ST 312: Ch. 12 Examples

Example 1. A large annuity company holds many industry group stocks. Among the industries are banks, business services and construction. Seven companies from each industry group are randomly sampled to test the hypothesis that the mean price per share is the same among industries. The data are:

Banks (Group 1)	Business Services (Group 2)	Construction (Group 3)
$n_1 = 7$ $\bar{x}_{.1} = 41$	$n_2 = 7$ $\bar{x}_{.2} = 30$	$n_3 = 7$ $\bar{x}_{.3} = 25$

(a) Complete the ANOVA table

Source	SS (Sums of Squares)	df (degrees of freedom)	MS (Mean of squares)	F statistic
Between Groups	938	$k-1 = 2$	$\frac{938}{2} = 469$	$\frac{469}{151.67} = 3.09$
Within Groups **(Error)**	$3668 - 988 = 2730$	$n_T - k = 18$	$\frac{2730}{18} = 151.67$	
Total	3668	$n_T - 1 = 20$		

(b) Conduct the 5-step ANOVA with $\alpha = 0.1$ to see if the industry group stocks differ among different industries.

(i) State the null and alternative hypothesis.

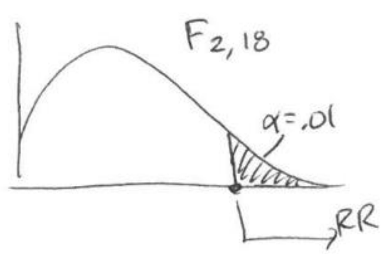
$$H_0: \mu_1 = \mu_2 = \mu_3$$
$$H_A: at\ least\ one\ \mu_i\ differs$$

(ii) $\alpha = 0.1$

(iii) Report the test statistic

$$F^* = 3.09\ \ with\ \ df_1 = 2, \quad df_2 = 18$$

(iv) Specify the rejection region



$$RR = \{F^* > F_{2,18,0.01} = 2.62\}$$

(v) Draw conclusion. Write a formal interpretation for the decision.

Since F* is in the RR, Reject the null. There is sufficient evidence at $\alpha = 0.05$ to suggest that mean price per share is not equal among industries.

(c) Is a post-hoc ANOVA analysis required here? Yes No (circle one)

(d) Specify the contrast (denoted as ψ_1) for comparing the **bank** and **business services**. Write this contrast without fractions.

$$\psi_1 = \mu_1 - \mu_2$$

(e) Specify the contrast (denoted as ψ_2) for comparing the mean stock difference between **construction** and the average of **bank** and **business services**. Write this contrast without fractions.

$$\psi_2 = \mu_3 - \frac{\mu_1 + \mu_2}{2} = 2\mu_3 - \mu_1 - \mu_2$$

(f) Report the estimate and the SE for ψ_2.

$$\hat{\psi}_2 = c_2 = 2\bar{x}_3 - \bar{x}_1 - \bar{x}_2 = 2(25) - 41 - 30 = -21$$

$$\hat{\sigma}_{c_2} = \sqrt{MSE * \sum_{i=1}^{k} \frac{a_i^2}{n_i}} = \sqrt{151.67 * \left(\frac{2^2}{7} + \frac{(-1)^2}{7} + \frac{(-1)^2}{7}\right)} = 11.402$$

(g) Test if the contrasts are equal to 0 by completing the table below using $\alpha_E = 0.1$. Assume the p-value given in the table for ψ_1 is correct.

What is the threshold of p-value for each comparison based on Bonferroni procedure? $\alpha_I = \frac{0.1}{2} = 0.05$

Hypotheses	Estimate (c)	SE of c	Test Statistics t	df	p-value	Reject H_0?
$H_0: \psi_1 = 0$ $H_A: \psi_1 \neq 0$	No need to compute	No need to compute	No need to compute	18	0.35	No (p > 0.05)
$H_0: \psi_2 = 0$ $H_A: \psi_2 \neq 0$	$c_2 = -21$	$\hat{\sigma}_{c_2} = 11.402$	$t = \frac{-21 - 0}{11.402} = -1.84$	18	$= 2P(t_{18} > 1.84) = 0.08$	No (p > 0.05)

(h) Practical conclusion based on (g)

There is no significant difference between means of bank and business stocks. There is no significant difference between the mean of construction and the average of bank and business stocks.

Example 2. True or False: In a one-way ANOVA

___**F**_____ MSE is a measure of ~~between~~-samples variation.

___**T**_____ MSE is a measure of within-samples variation.

___**T**_____ MSG is a measure of between-samples variation.

(MSE)

___**F**_____ MSG provides a valid estimate of the common variance regardless H_0 is true or false.

Example 3. The Kenton Food Company wished to test four different package designs for a new breakfast cereal. Twenty stores, with approximately equal sales volumes, were selected as the experimental unit. Each store was randomly assigned one of the package designs, with each package design assigned to five stores. The stores were chosen to be comparable in location and sales volume. Other relevant conditions that could affect sales, such as price, amount and location of shelf space, and special promotional efforts were kept the same for all of the stores in the experiment.

Package Design	n	Mean	SD
A	5	14.6	2.3022
B	5	13.4	3.6469
C	5	19.4	2.3022
D	5	27.2	3.9623

(a) Write the hypotheses for the ANOVA test to determine if the mean number of cases sold is the same for each package design.

$$H_0: \mu_A = \mu_B = \mu_C = \mu_D$$

$$H_A: at\ least\ one\ \mu_i\ differs$$

(b) Check the conditions of the test using the summary statistics provided above.

(1) Must assume Normality (no information given), (2) Can assume independence based on description of experiment (randomly assigned), (3)The smallest SD is within twice the largest (variances are roughly equal).

(c) Complete the ANOVA table below:

Source of Variation	*SS*	*df*	*MS*	*F*
Between Groups	588.15	**3**	**196.05**	**19.8**
Within Groups	**158.4**	**16**	**9.9**	
Total	746.55	19		

(d) Is there evidence at the 5% significance level to suggest a difference in mean sales?

$$RR = \{F^* > F_{3,16,0.05} = 3.24\}$$

Since F* = 19.8 is in the RR, we can reject the null hypothesis.

There is enough evidence at the 5% significance level to suggest that the package design affects mean sales (a significant difference exists).

(e) Below is output from a multiple comparison test using the Bonferroni procedure. In this output, we are given adjusted p-values for each pair of means (compare each to 0.05). What can we conclude about the package designs?

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Pairwise comparisons using t tests with pooled SD

  A      B      C
B 1.0000 -      -
C 0.1694 0.0493 -
D 6e-05  2e-05  0.0073

P value adjustment method: bonferroni
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The above p-values are less than 0.05, so we can conclude that the mean sales for package design D differ from all others (A & D, B & D, C & D), and that designs B & C differ.

Example 4. A chemical engineer wants to compare the hardness of four blends of paint. Six samples of each paint blend were applied to a piece of metal. The pieces of metal were cured. Then each sample was measured for hardness. In order to test for the equality of means and to assess the differences between pairs of means, the analyst uses one-way ANOVA. Summary statistics and the ANOVA table are provided below.

Paint	n	Mean
Blend 1	6	14.73
Blend 2	6	8.57
Blend 3	6	12.98
Blend 4	6	18.07

Source	df	SS	MS	F
Paint	3	281.7	93.9	6.02
Error	20	312.1	15.6	
Total	23	593.8		

(a) Can we reject the one-way ANOVA null hypothesis at the 1% significance level?

$RR = \{F^* > F_{3,20,0.01} = 4.94\}$

Since F* = 6.02 is in the RR, we can reject the null hypothesis.

There is enough evidence at the 1% significance level to suggest that the mean hardness differs across blends.

(b) Is there evidence that Blend 2 is significantly different from the mean of the other Blends? Keep the family-wise error rate the same as in (a).

$$\psi = \mu_2 - \frac{\mu_1 + \mu_3 + \mu_4}{3} = 3\mu_2 - \mu_1 - \mu_3 - \mu_4$$

$$H_0: \psi = 0$$

$$H_A: \psi \neq 0$$

$$\hat{\psi} = c = 3\bar{x}_2 - \bar{x}_1 - \bar{x}_3 - \bar{x}_4 = -20.07$$

$$\hat{\sigma}_c = \sqrt{15.6 * \left(\frac{3^2}{6} + \frac{(-1)^2}{6} + \frac{(-1)^2}{6} + \frac{(-1)^2}{6}\right)} = 5.5857$$

$$t^* = \frac{-20.07 - 0}{5.5857} = -3.59 \; with \; df = 20$$

$$p = 2 * P(t_{20} > 3.6) = 0.002$$

Since $p < \alpha$, Reject H_0. There is evidence at the 1% significance level to suggest that blend 2 is different from the mean of the other blends.