

ST 312: Ch. 12 Examples

Example 1. A large annuity company holds many industry group stocks. Among the industries are banks, business services and construction. Seven companies from each industry group are randomly sampled to test the hypothesis that the mean price per share is the same among industries. The data are:

| Banks (Group 1) | Business Services (Group 2) | Construction (Group 3) |
|------------------------|------------------------------------|-------------------------------|
| $n_1 = 7$ | $n_2 = 7$ | $n_3 = 7$ |
| $\bar{x}_1 = 41$ | $\bar{x}_2 = 30$ | $\bar{x}_3 = 25$ |

(a) Complete the ANOVA table

| Source | SS (Sums of Squares) | df (degrees of freedom) | MS (Mean of squares) | F statistic |
|------------------------------------|-------------------------------------|--|---------------------------------|--------------------|
| Between Groups | 938 | | | |
| Within Groups (Error) | | | | |
| Total | 3668 | | | |

(b) Conduct the 5-step ANOVA with $\alpha = 0.1$ to see if the industry group stocks differ among different industries.

(i) State the null and alternative hypothesis.

(ii) $\alpha = 0.1$

(iii) Report the test statistic

(iv) Specify the rejection region

(v) Draw conclusion. Write a formal interpretation for the decision.

(c) Is a post-hoc ANOVA analysis required here? Yes No (circle one)

(d) Specify the contrast (denoted as ψ_1) for comparing the **bank** and **business services**. Write this contrast without fractions.

(e) Specify the contrast (denoted as ψ_2) for comparing the mean stock difference between **construction** and the average of **bank** and **business services**. Write this contrast without fractions.

(f) Report the estimate and the SE for ψ_2 .

(g) Test if the contrasts are equal to 0 by completing the table below using $\alpha_E = 0.1$. Assume the p-value given in the table for ψ_1 is correct.

What is the threshold of p-value for each comparison based on Bonferroni procedure? _____

| Hypotheses | Estimate (c) | SE of c | Test Statistics t | df | p-value | Reject H_0 ? |
|---|--------------------|--------------------|---------------------|----|---------|----------------|
| $H_0: \psi_1 = 0$ $H_A: \psi_1 \neq 0$ | No need to compute | No need to compute | No need to compute | | 0.35 | |
| $H_0: \psi_2 = 0$ $H_A: \psi_2 \neq 0$ | | | | | | |

(h) Practical conclusion based on (g)

Example 2. True or False: In a one-way ANOVA

- _____ MSE is a measure of between-samples variation.
- _____ MSE is a measure of within-samples variation.
- _____ MSG is a measure of between-samples variation.
- _____ MSG provides a valid estimate of the common variance regardless H_0 is true or false.

Example 3. The Kenton Food Company wished to test four different package designs for a new breakfast cereal. Twenty stores, with approximately equal sales volumes, were selected as the experimental unit. Each store was randomly assigned one of the package designs, with each package design assigned to five stores. The stores were chosen to be comparable in location and sales volume. Other relevant conditions that could affect sales, such as price, amount and location of shelf space, and special promotional efforts were kept the same for all of the stores in the experiment.

| Package Design | n | Mean | SD |
|----------------|---|------|--------|
| A | 5 | 14.6 | 2.3022 |
| B | 5 | 13.4 | 3.6469 |
| C | 5 | 19.4 | 2.3022 |
| D | 5 | 27.2 | 3.9623 |

- (a) Write the hypotheses for the ANOVA test to determine if the mean number of cases sold is the same for each package design.

- (b) Check the conditions of the test using the summary statistics provided above.

- (c) Complete the ANOVA table below:

| <i>Source of Variation</i> | <i>SS</i> | <i>df</i> | <i>MS</i> | <i>F</i> |
|----------------------------|-----------|-----------|-----------|----------|
| Between Groups | 588.15 | | | |
| Within Groups | | | | |
| | | | | |
| Total | 746.55 | 19 | | |

- (d) Is there evidence at the 5% significance level to suggest a difference in mean sales?

(e) Below is output from a multiple comparison test using the Bonferroni procedure. In this output, we are given adjusted p-values for each pair of means (compare each to 0.05). What can we conclude about the package designs?

Pairwise comparisons using t tests with pooled SD

```

      A      B      C
B 1.0000 -      -
C 0.1694 0.0493 -
D 6e-05 2e-05 0.0073

```

P value adjustment method: bonferroni

Example 4. A chemical engineer wants to compare the hardness of four blends of paint. Six samples of each paint blend were applied to a piece of metal. The pieces of metal were cured. Then each sample was measured for hardness. In order to test for the equality of means and to assess the differences between pairs of means, the analyst uses one-way ANOVA. Summary statistics and the ANOVA table are provided below.

| Paint | n | Mean |
|---------|---|-------|
| Blend 1 | 6 | 14.73 |
| Blend 2 | 6 | 8.57 |
| Blend 3 | 6 | 12.98 |
| Blend 4 | 6 | 18.07 |

| Source | df | SS | MS | F |
|--------|----|-------|------|------|
| Paint | 3 | 281.7 | 93.9 | 6.02 |
| Error | 20 | 312.1 | 15.6 | |
| Total | 23 | 593.8 | | |

(a) Can we reject the one-way ANOVA null hypothesis at the 1% significance level?

(b) Is there evidence that Blend 2 is significantly different from the mean of the other Blends? Keep the family-wise error rate the same as in (a).