ST 312 Ch. 7.2 Practice

1. Consider an experiment involving the comparison of the mean heart rate following 30 minutes of aerobic exercise among females aged 20 to 24 years as compared to females aged 30-34 years. For this experiment, 10-second heart rates are recorded on each participant following 30 minutes of intense aerobic exercise and converted to beats per minute (i.e., heart rate per 60 seconds). The sample data are given below:

	Age 20-24	Age 30-34
Sample size	15	10
Sample mean	150.22	141.10
Sample SD	40.0	10.0

Assume the population variances of heart rate are different for these two age groups, and $df = 16.522$.

(a) Calculate the 95% confidence interval of the mean difference.

Need $t_{\alpha/2}: t_{0.025,16} = 2.120$

95% CI for : $\mu_1 - \mu_2$**:**

$$(150.22 - 141.10) \pm 2.120\sqrt{\frac{40^2}{15} + \frac{10^2}{10}} = (-13.776, 32.016)$$

We are 95% confident that the difference in the mean heart rates of females age 20-24 and females age 30-34 is between -13.776 and 32.016 beats per minute.

(b) How would you explain your results?

We could suggest that that difference in the mean heart rates between the age groups **(is /** ***is not*****)** significantly different from 5 bpm because the 95% CI for $\mu_1 - \mu_2$ ____**contains 5**_______________.

(c) In the notes, we did a hypothesis test using this data. Explain how the results of these two analyses differ.

The hypothesis test was a one-sided test and allowed us to determine if the difference was greater than 5. The confidence interval allows us to determine if the difference is different from (i.e. not equal to) 5, since a confidence interval is equivalent to a two-sided test.

2. The concentration of benzene was measured in units of milligrams per liter for simple random samples of specimens of wastewater produced at a gas field. The summary statistics are given in the table below. Assuming the populations are Normal, can you conclude that the mean benzene concentration is less in treated water than in untreated water? Use a 5% significance level (do not assume the variances are equal).

	Untreated wastewater	Treated wastewater
n	5	7
$\bar{x}$	7.8	3.2
s	1.4	1.7

Is $\mu_{treated} < \mu_{untreated}$? That is, is $\mu_2 < \mu_1$, or $\mu_1 - \mu_2 > 0$?

$$H_0: \mu_1 - \mu_2 \leq 0$$
$$H_A: \mu_1 - \mu_2 > 0$$
$$\alpha = 0.05$$
$$t^* = \frac{7.8 - 3.2 - 0}{\sqrt{\frac{1.4^2}{5} + \frac{1.7^2}{7}}} = 5.12$$

$$df = \nu = \frac{\left(\frac{1.4^2}{5} + \frac{1.7^2}{7}\right)^2}{\frac{\left(\frac{1.4^2}{5}\right)^2}{5-1} + \frac{\left(\frac{1.7^2}{7}\right)^2}{7-1}} = 9.694 \approx 9$$

P-value: $p = P(t_9 > 5.12) = 0$

Since $p < \alpha, Reject\ H_0$. There is enough evidence at $\alpha = 0.05$ to suggest that the mean benzene concentration is less in treated water than in untreated water.

3. Is there a difference between the life of batteries (measured in days) made by Duracell and Eveready? Summary statistics from two random samples are given below. Assume the populations are Normal. Calculate and interpret a 99% confidence interval for the difference in mean lifetime.
Do not assume equal variances. Let $\nu = 15$.

	Duracell	Eveready
Sample size	8	10
Sample mean	41	45
Sample SD	18	20

Need $t_{\alpha/2}$: $t_{0.005,15} = 2.947$
99% CI for : $\mu_1 - \mu_2$:

$$(41 - 45) \pm 2.947\sqrt{\frac{18^2}{8} + \frac{20^2}{10}} = (-30.441, 22.441)$$

We are 99% confident that the difference in the mean lifetimes of batteries made by Duracell and Eveready is between -30.441 and 22.441 days.

4. A sample of 37 subjects went on a low-carb diet for six months and the mean weight loss for this group was 4.7 kilograms with a standard deviation of 7.16 kilograms. A different sample of 29 subjects went on a low-fat diet and the mean weight loss for this group was 3.6 kilograms with a standard deviation of 6.9 kilograms.
   a. Assuming the populations are Normal and variances are **equal**, find a 95% confidence interval for the difference of means between low-carb and low-fat diet weight loss.

Low-Carb	**Low-Fat**
***m* = 37**	***n* = 29**
$\overline{x} = 4.7$	$\overline{x} = 3.6$
$s_1 = 7.16$	$s_2 = 6.9$

Need df, $t_{\alpha/2}$, and s_p^2.

df = m + n – 2 = 64

$t_{0.025,64} = 1.998$

$$s_p^2 = \frac{(37-1)(7.16)^2 + (29-1)(6.9)^2}{64} = 49.6663$$

95% CI for : $\mu_1 - \mu_2$: $(4.7 - 3.6) \pm 1.998\sqrt{49.6663 * \left(\frac{1}{37} + \frac{1}{29}\right)} = (-2.3922, 4.5922)$

   b. Can we conclude the mean weight loss is different for these two diets? Explain.

Since the interval contains 0, we cannot conclude that the mean weight loss for these diets is significantly different.

5. A local pizza restaurant is close to a college campus. They advertise that their delivery is faster than the local competitor of a national chain to the college dormitory. Ten statistics students (who all live in the dorms) decide to test this claim. Each chooses a time and calls both restaurants at the same time and records the delivery time of both. The results of the delivery time (in minutes) is summarized in the table below. Is there evidence at a 0.01 level that the local store does deliver to the dormitory faster? Assume the difference in delivery times is normally distributed. Let *d* = *local – chain*.

	Local	Chain	Difference
Sample mean	16.7	18.88	-2.18
Sample SD	3.096	2.866	2.264

Is local faster? "Faster" means a smaller delivery time, so if *local < chain,* then *d = local – chain* would be negative.

$H_0: \mu_D \geq 0$

$H_A: \mu_D < 0$

$\alpha = 0.01$

$n = 10, df = 9$

$$t^* = \frac{-2.18 - 0}{2.264/\sqrt{10}} = -3.04$$

P-value: $p = P(t_9 < -3.04) = 0.007$

Since $p < \alpha, Reject\ H_0$**. There is enough evidence at** $\alpha = 0.01$ **to suggest that, on average, the delivery time of the local store is faster than that of the chain store.**

6. A particular SAT Prep Course claims to improve SAT verbal scores by 10 points and math scores by 20 points. To test this claim, twenty students take the SAT, take the SAT Prep Course, then retake the SAT. When subtracting $score_{after\ course} - score_{before\ course}$, the mean of the differences is 28 and the standard deviation is 26.5. Does this data support the claim that students who take the course improve their SAT scores by an average of at least 30 points when α = 0.05? Assume that the differences in scores are normally distributed.
Is the improvement at least 30 points? That is, is $\mu_D \geq 30$?

$H_0: \mu_D \geq 30$

$H_A: \mu_D < 30$

$\alpha = 0.05$

$n = 20, df = 19$

$$t^* = \frac{28 - 30}{26.5/\sqrt{20}} = -0.33$$

P-value: $p = P(t_{19} < -0.33) = 0.3725$

Since $p > \alpha$, $Fail\ to\ Reject\ H_0$. **There is not enough evidence at** $\alpha = 0.05$ **to reject the claim that the students who take the course improve their SAT scores by an average of at least 30 points.**

***Note: We cannot *support* this claim, nor *any* claim that is in the null hypothesis!**