ST 312 Ch. 8.2 Practice

1. Voters in a particular city who identify themselves with one or the other of two political parties were randomly selected and asked if they favor a proposal to allow citizens with proper license to carry a concealed handgun in city parks. The samples included 150 voters from Party A and 200 voters from Party B. The numbers in favor of the proposal were 90 from Party A and 140 from Party B. Conduct a test to determine if the parties significantly differ in their support of the proposal. Use α = 0.05.

   Let $p_1 =$ proportion from Party A who favor the proposal and $p_2 =$ proportion from Party B who favor the proposal.

   $$H_0: p_1 - p_2 = 0$$
   $$H_A: p_1 - p_2 \neq 0$$
   $$\alpha = 0.05$$
   $$\hat{p}_1 = \frac{90}{150} = 0.60$$
   $$\hat{p}_2 = \frac{140}{200} = 0.70$$
   $$m\hat{p}_1, m\hat{q}_1, n\hat{p}_2, n\hat{q}_2 \geq 10$$
   $$\hat{p} = \frac{90 + 140}{150 + 200} = 0.6571$$

   $$z^* = \frac{0.60 - 0.70 - 0}{\sqrt{0.6571 * 0.3429\left(\frac{1}{150} + \frac{1}{200}\right)}} = -1.95$$

   P-value: $p = 2P(Z < -1.95) = 2(0.0256) = 0.0512$

   Since $p > \alpha$, $FTR\ H_0$. There is not enough evidence at $\alpha = 0.05$ to suggest a significant difference exists in the proportions of Party A and Party B members who favor the proposal.

2. A method of analyzing digital music files to determine the key in which the music was written was tested to determine its accuracy. In a sample of 307 pop music selections, the key was correctly identified in 245 of them. In a sample of 347 new age sections, the key was correctly identified in 304 of them. Calculate a 99% confidence interval for the difference in the proportions of correctly identified keys in pop and new age music. What can we conclude?

$m = 307, \hat{p}_1 = \frac{245}{307} = 0.7980$

$n = 347, \hat{p}_2 = \frac{304}{347} = 0.8761$

$m\hat{p}_1, m\hat{q}_1, n\hat{p}_2, n\hat{q}_2 \geq 10$

$z_{\frac{\alpha}{2}} = z_{0.005} = 2.575$

99% CI for $p_1 - p_2$:

$$(0.7980 - 0.8761) \pm 2.575 \sqrt{\frac{0.7980 * 0.2020}{307} + \frac{0.8761 * 0.1239}{347}}$$

$$-0.0781 \pm 0.0745$$

$$(-0.1526, -0.0036)$$

We are 99% confident that the difference in the proportions of correctly identified keys in pop and new age music is between -0.1526 and -0.0036. Since 0 is not in this interval, we conclude that these proportions are significantly different.

3. We want to conduct a study to estimate the difference in the proportions of teenagers and adults who use TicTok. We want our estimate to have 90% confidence and a margin of error within 0.08. What sample size is required to meet these specifications?

Since no estimates for p_1 and p_2 are given,

$$n = \left(\frac{z_{\frac{\alpha}{2}}}{MOE}\right)^2 (p_1^* * q_1^* + p_2^* * q_2^*) = \left(\frac{1.645}{0.08}\right)^2 (0.5 * 0.5 + 0.5 * 0.5) = 211.41$$

We would need 212 from each population.