ST 312 Ch. 9 Practice

1. When a police officer responds to a call for help in a case of spousal abuse, what should the officer do? A randomized controlled experiment in Charlotte, North Carolina, studied three police responses to spousal abuse: advise and possibly separate the couple, issue a citation to the offender, and arrest the offender. The effectiveness of the three responses was determined by re-arrest rates (the number of subsequent arrests). The table below shows these rates. Based on the 650 cases, are the proportions of subsequent arrests the same regardless of the treatment assigned?

	Assigned treatment		
Number of subsequent arrests	Arrest	Citation	Advise/separate
0	175	181	187
1	36	33	24
2	2	7	1
3	1	1	0
4	0	2	0

A. Under the null hypothesis, what is the expected case counts for 4 re-arrests with the treatment "Advise/separate"?

$$E_{53} = \frac{212 * 2}{650} = 0.652$$

B. __ii_____Which of the following statements is true?

i. We cannot use a chi-square test because the police officers did not record the expected counts.

ii. We cannot use a chi-square test because the expected cell counts are less than five in too many of the cells.

iii. The test will have a very small P-value (below 0.0001) because there were so few cases where there was more than one re-arrest

C. Further combine the data into a 2x3 table, such that Subsequent Arrests are categorized as None or Yes (1 or more).

i. Complete the table below and conduct the chi-square test of independence at $\alpha = 0.1$.

Subsequent Arrests \ Assigned Tx		Arrest	Citation	Advise/Separate
None	Observed	175	181	187
	Expected	**178.77**	**187.13**	**177.1**
	χ^2 value	**0.08**	**0.20**	**0.55**
Yes	Observed	39	43	25
	Expected	**35.23**	**36.87**	**34.90**
	χ^2 value	**0.40**	**1.02**	**2.81**

H_0: $Re-arrest$ & $Treatment\ are\ independent$

H_A: $Re-arrest$ & $Treatment\ are\ dependent$

$\alpha = 0.10$

$\chi^2 = 0.08 + 0.20 + 0.55 + 0.40 + 1.02 + 2.81 = 5.06$

$with\ df = (2-1)(3-1) = 2$

$RR = \{\chi^{2^*} > \chi^2_{2,0.1} = 4.61\}$

Since $\chi^{2^*} = 5.06$ is in the RR, we Reject H_0. There is evidence at the 10% significance level to suggest an association exists between subsequent arrests and treatment.

ii. Roughly speaking, which cell(s) have larger contribution to the chi-square test statistics?

The largest is the "Advise/Separate" treatment with re-arrest (at 2.81), followed by the "Citation" treatment with re-arrest (at 1.02).

2. It is hypothesized that when homing pigeons are disoriented in a certain manner, they will exhibit no preference for any direction of flight after takeoff (so that the direction *X* should be uniformly distributed on the interval from 0° to 360°). To test this, 120 pigeons are disoriented, let loose, and the direction of flight of each is recorded; the resulting data follows.

A. Write a set of hypotheses to test whether the direction of flight after takeoff is uniform.

H_0: Directions of flight have equal proportions (no preference)

H_A: Some direction of flight is more or less likely than others (some preference)

This may also be written:

$H_0: All\ p_i = \frac{1}{8}$

H_A: at least one p_i is different

B. Use the Chi-Squared Goodness of Fit test at level of 0.10 to see whether the data supports the hypothesis from part (a).

Direction	Frequency	Expected	χ^2
[0, 45)	12	$120\left(\frac{1}{8}\right) = 15$	$\frac{(12-15)^2}{15} = 0.6$
[45, 90)	16	$120\left(\frac{1}{8}\right) = 15$	$\frac{(16-15)^2}{15} = 0.067$
[90, 135)	17	$120\left(\frac{1}{8}\right) = 15$	$\frac{(17-15)^2}{15} = 0.267$
[135, 180)	15	$120\left(\frac{1}{8}\right) = 15$	$\frac{(15-15)^2}{15} = 0$
[180, 225)	13	$120\left(\frac{1}{8}\right) = 15$	$\frac{(13-15)^2}{15} = 0.267$
[225, 270)	20	$120\left(\frac{1}{8}\right) = 15$	$\frac{(20-15)^2}{15} = 1.67$
[270, 315)	17	$120\left(\frac{1}{8}\right) = 15$	$\frac{(17-15)^2}{15} = 0.267$
[315, 360)	10	$120\left(\frac{1}{8}\right) = 15$	$\frac{(10-15)^2}{15} = 1.67$
			$\chi^2 = 4.8, df = 7$

$RR = \{\chi^{2*} > \chi^2_{7,0.1} = 12.02\}$

Since 4.8 is not in the RR, we Fail to Reject H_0. There is not enough evidence at the 10% significance level to reject the hypothesis that homing pigeons exhibit no preference for direction of flight after takeoff.

3. A poll conducted by the General Social Survey asked 1155 people whether they thought that people with high incomes should pay a greater or smaller percentage of their income in tax than low-income people. The results are presented in the following table.

Category	Observed
Pay much more	218
Pay somewhat more	497
Pay the same	425
Pay less	15

Five years earlier, it was determined that 18.5% believed that the rich should pay much more, 39.2% believed they should pay somewhat more, 41.2% believed they should pay the same, and 1.1% believed they should pay less. Can we conclude that the current percentages differ from these? Use α = 0.05.

$$H_0: p_1 = 0.185, p_2 = 0.392, p_3 = 0.412. p_4 = 0.011$$

$$H_A: some\ of\ the\ probabilities\ are\ not\ equal\ to\ those\ specified\ in\ H_0$$

$$\alpha = 0.05$$

Category	Observed	Expected	χ^2
Pay much more	218	$np_1 = 1155(0.185) = 213.675$	$\frac{(218 - 213.675)^2}{213.675} = 0.0875$
Pay somewhat more	497	$np_2 = 1155(0.392) = 452.76$	$\frac{(497 - 452.76)^2}{452.76} = 4.3228$
Pay the same	425	$np_3 = 1155(0.412) = 475.86$	$\frac{(425 - 475.86)^2}{475.86} = 5.4359$
Pay less	15	$np_4 = 1155(0.011) = 12.705$	$\frac{(15 - 12.705)^2}{12.705} = 0.4146$
			$\chi^2 = 10.261, df = 3$

$$RR = \{\chi^{2*} > \chi^2_{3,0.05} = 7.815\}$$

Since 10.261 is in the RR, we Reject H_0. There is enough evidence at the 5% significance level to suggest that opinions on this issue have changed during the 5 years prior to the survey.

4. A recent study examined the effects of carbon monoxide exposure on a group of construction workers. The following able presents the numbers of workers who reported various symptoms, along with the shift (morning, evening, or night) that they worked.

Symptom \ Shift	Morning	Evening	Night	
Influenza	16	13	18	
Headache	24	33	6	**47**
Weakness	11	16	5	**63**
Shortness of Breath	7	9	9	**32**
	58	**71**	**38**	**167**

Can you conclude that symptom and shift are not independent? Use $\alpha = 0.05$.

H_0: *Symptom and Shift are independent*

H_A: *Symptom and Shift are dependent*

$\alpha = 0.05$

Expected Cell Counts:

Symptom \ Shift	Morning	Evening	Night
Influenza	$\frac{47*58}{167}$ =16.323	$\frac{47*71}{167}$ =19.982	10.695
Headache	21.880	26.784	14.335
Weakness	11.114	13.605	7.281
Shortness of Breath	8.683	10.629	5.689

$$\chi^2 = \frac{(16-16.323)^2}{16.323} + \frac{(24-21.880)^2}{21.880} + \cdots + \frac{(5-7.281)^2}{7.281} + \frac{(9-5.689)^2}{5.689} = 17.572$$

$$with\ df = (4-1)(3-1) = 6$$

$$RR = \{\chi^{2^*} > \chi^2_{6,0.05} = 12.59\}$$

Since $\chi^{2*} = 17.572$ is in the RR, we Reject H_0. There is evidence at the 5% significance level to suggest an association exists between symptom and shift (i.e. they are dependent).