Confidence Interval for a Proportion

These notes present the method for finding a confidence interval estimate of a population proportion.

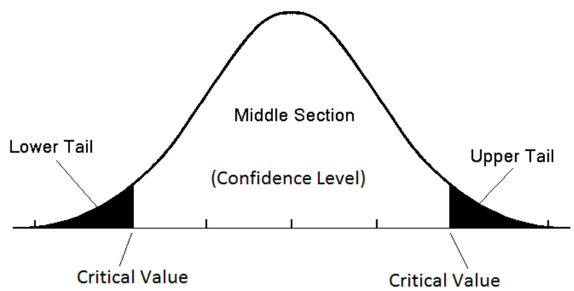
The sample proportion $\hat{p}$ is the best point estimate of the population proportion p. If the following conditions are met

1. The sample is a simple random sample.
2. $np \geq 10$ and $nq \geq 10$

then we can use the Normal distribution as a suitable approximation to the binomial distribution.

Finding a Critical Value

The confidence level is incorporated into the confidence interval through a value known as the *critical value*. The confidence level is the area under the distribution curve that is between the critical values.



To find a critical value from a Normal distribution, you need to know the confidence level. Our table allows us to find the area to the left of a z-score. So you need to use the graph above and the confidence level to determine the appropriate cumulative area.

<u>Example</u>: A sample of size n = 80 is a simple random sample. We count the number of these 80 who are right-handed and use this sample proportion to calculate a confidence interval for the true proportion of right-handed people. Find the critical value z_c corresponding to a 95% confidence level.

> The confidence level is 95%, so there is 5% remaining. Divide it by 2, leaving 2.5% in each tail. The cumulative area corresponding to the positive critical value is 0.95 + 0.025 = 0.975. Find the z-score corresponding to a cumulative area of 0.975 using the table.
> $z_c = 1.96$

Margin of Error

Recall that the margin of error, E, is found by multiplying the critical value and the standard error.

$$E = z_c\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

Given the way that the margin of error E is defined, there is a probability of 1 – C that the sample proportion will be in error by more than E.

Procedure for Constructing a Confidence Interval for p

1. Verify that the requirements are satisfied. (We have a simple random sample, and both $np \geq 10$ and $nq \geq 10$)

2. Find the critical value z_c that corresponds to the desired confidence level.

3. Evaluate the margin of error

$$E = z_c\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

4. Using the value of the calculated margin of error E and the value of the sample proportion $\hat{p}$, find the values of the interval limits:

$$\hat{p} \pm E \text{ or } (\hat{p} - E,\ \hat{p} + E)$$

   Round the resulting confidence interval limits. In general, for proportions, use 4 decimals.

5. Interpret the Confidence Interval.

Example

Many modern psychologists and therapists encourage the use of mindfulness, in the form of mindfulness-based cognitive therapy (MCBT), a non-pharmacological treatment for issues such as anxiety and depression. 145 individuals who were in remission or recovery for major depression were treated with MCBT after being randomly selected for the study. 57 of the patients relapsed into major depression at some point in the two years following treatment. Create a 90% confidence interval for the true proportion of all patients recovering from major depression treated with MCBT that will relapse into major depression in the two years following treatment. Round your final answer to 3 places. Interpret your confidence interval.

Solution:

The 90% Confidence interval is given by

$$\left(\hat{p} - z_c\sqrt{\frac{\hat{p}\hat{q}}{n}}, \hat{p} + z_c\sqrt{\frac{\hat{p}\hat{q}}{n}}\right)$$

where $\hat{p} = \frac{57}{145} = 0.393$, $z_{0.95} = 1.645$, and $\sqrt{\frac{\hat{p}\hat{q}}{n}} = \sqrt{\frac{0.393*(1-0.393)}{145}} = 0.0406$.

We get:

$$(0.393 - 1.645 * 0.0406, 0.393 + 1.645 * 0.0406) = (0.326, 0.460)$$

Interpretation: We are 90% confident that the true proportion of all patients recovering from major depression treated with MCBT that will relapse into major depression is between 0.326 and 0.460.