ST 312: Midterm 2 Review Problems

Part I: Multiple Choice, True/False Questions

1. When is the Satterthwaite approximation for the degrees of freedom necessary?
   a. When the population variances are unknown
   b. When the population variances are unequal
   c. When the population variances are equal
   d. When the data are paired
   e. When the sample variances are unequal

2. A pooled variance estimator is used when…
   a. The data is paired
   b. Calculating the sample size
   c. The null hypothesis is true
   d. The variances of two populations are assumed equal
   e. The variances of two populations are unequal

3. A pooled proportion estimator is used when…
   a. The sample proportions are unknown
   b. Finding the minimum sample size
   c. Conducting a hypothesis test for $p_1 - p_2$
   d. Constructing a confidence interval for $p_1 - p_2$
   e. All of the above

4. The Normal distribution can be used as the sampling distribution under the null hypothesis for a test concerning $p_1 - p_2$ if…
   a. The sample sizes (n_1, n_2) are larger than 30
   b. The distributions of X_1 and X_2 are binomial
   c. Either $n_1p_1, n_1q_1 \geq 10$ or $n_2p_2, n_2q_2 \geq 10$
   d. Both $n_1p_1, n_1q_1 \geq 10$ and $n_2p_2, n_2q_2 \geq 10$

5. The Normal distribution can be used as the sampling distribution under the null hypothesis for a test concerning $\mu_1 - \mu_2$ if…
   a. The sample sizes (n_1, n_2) are larger than 30
   b. The distributions of X_1 and X_2 are Normal
   c. The variances (σ_1, σ_2) are known
   d. Both *a* and *c* above
   e. Either *a* or *b*, and *c* above

6. A Chi-Square statistic can be used to test for independence when…
    a. The sample size is at least 30
    b. The observed cell counts are all 5 or greater
    c. The expected cell counts are all 5 or greater
    d. The test statistic is larger than the critical value

7. A hypothesis test to determine if the mean of population A is larger than the mean of population B finds a test statistic of 1.85. Interpret this value.
    a. The difference between μ_A and μ_B is 1.85.
    b. The difference between $\bar{x}_A$ and $\bar{x}_B$ is 1.85.
    c. If the population means were equal, the difference between $\bar{x}_A$ and $\bar{x}_B$ lies 1.85 standard deviations above 0.
    d. If the population means are both 0, the difference between $\bar{x}_A$ and $\bar{x}_B$ lies 1.85 standard deviations above 0.

8. A hypothesis test to determine if the mean difference of treatment A and treatment B is greater than 0 results in a p-value of 0.04. Interpret this value.
    a. The probability that the null hypothesis is true is 0.04.
    b. The probability that the mean difference is greater than 0 is 0.04.
    c. The probability of observing a mean difference equal to or greater than the sample mean difference is equal to 0.04.
    d. The probability of observing a mean difference equal to or greater than the sample mean difference, assuming the true mean difference is 0, is equal to 0.04.
    e. The probability of observing a mean difference equal to or greater than 0, assuming the true mean difference is greater than or equal to than the sample mean difference, is equal to 0.04.

9. The *t* critical value for a 95% confidence interval estimation with 24 degrees of freedom is
    a. 1.71
    b. 2.064
    c. 2.492
    d. 2.069

10. The *z* critical value for a 99% confidence interval estimation is
    a. 1.645
    b. 1.96
    c. 2.33
    d. 2.58

11. The rejection region for a Chi-square test of independence where $\alpha = 0.05$ and the two-way table consists of 4 rows and 3 columns is
    a. $RR = \{\chi^{2*} < 11.07\}$
    b. $RR = \{\chi^{2*} > 12.59\}$
    c. $RR = \{\chi^{2*} < 12.59\}$
    d. $RR = \{\chi^{2*} > 14.07\}$
    e. $RR = \{\chi^{2*} > 21.03\}$
    f. $RR = \{\chi^{2*} < 21.03\}$

12. In a two-tailed hypothesis test the test statistic z* is determined to be -2.5. The p-value for this test is
    a. -1.25
    b. 0.4938
    c. 0.0062
    d. 0.0124

13. In a left-tailed hypothesis test, the test statistic z* is determined to be -2. The p-value for this test is
    a. 0.4772
    b. 0.0228
    c. 0.0056
    d. 0.0238

14. In a left-tailed hypothesis test, the test statistic t* is determined to be -2 with df = 9. The p-value for this test is
    a. 0.038
    b. 0.962
    c. 0.000
    d. 1.0

15. A test for paired differences uses which sampling distribution?
    a. A standard normal
    b. A *t* distribution with df = the total number of observations
    c. A *t* distribution with df = the total number of observations - 1
    d. A *t* distribution with df = the total number of differences
    e. A *t* distribution with df = the total number of differences – 1

16. The value added and subtracted from a point estimate in order to develop an interval estimate of the population parameter is known as the
    a. confidence level
    b. margin of error
    c. confidence coefficient
    d. parameter estimate
    e. interval estimate
    f. critical value

17. Which value(s) are used as an estimator(s) for the proportions p_1 and p_2 when calculating the standard error of $\hat{p}_1 - \hat{p}_2$ during a hypothesis test?
    a. $\hat{p}_1$
    b. $\hat{p}_2$
    c. Both $\hat{p}_1$ and $\hat{p}_2$
    d. $\hat{p}$

18. If a test rejects H_0: $\mu_1 = \mu_2$, then the confidence interval for ($\mu_1 - \mu_2$) having the same error rate does not contain zero.
    a. True
    b. False

19. If the calculated value of the *t* test statistic is negative, then there is strong evidence that the null hypothesis is false.
    a. True
    b. False

20. Expected cell counts are calculated the same way in the tests for independence and goodness-of-fit.
    a. True
    b. False

21. The Chi-square goodness-of-fit test always assumes the proportions to be equal under the null hypothesis.
    a. True
    b. False

22. A Chi-square test for independence returns a test statistic that is greater than the critical value. We can conclude that the variables are dependent.
    a. True
    b. False

23. A study is conducted to evaluate of the improvement in aerobic fitness for 15 subjects where measurements are made at the beginning of a fitness program and at the end of it. Measurements are made such that a higher aerobic score represents a greater aerobic fitness level. Which set of hypotheses is correct for this test? Let *d = ending aerobic score – beginning aerobic score*.

a. $H_0: \mu_d = 0$
   $H_A: \mu_d \neq 0$

b. $H_0: \mu_d \geq 0$
   $H_A: \mu_d < 0$

c. $H_0: \mu_d \leq 0$
   $H_A: \mu_d > 0$

d. $H_0: \mu_d > 0$
   $H_A: \mu_d \leq 0$

e. $H_0: \mu_d < 0$
   $H_A: \mu_d \geq 0$

24. The deterioration of pipeline networks across the country is a growing concern. One rehabilitation option proposed is to thread a liner through existing pipe. Wishing to know whether fusing liner to the pipes increases tensile strength, measurements were taken for 10 unfused liners and 8 fused liners. A 95% confidence interval for $\mu_{No\ fusion} - \mu_{fusion}$ was found to be $(-488, 38)$. We can conclude:

a. There is no significant difference in tensile strength between fused and unfused liners because 0 is in the interval.
b. There is a significant difference in tensile strength between fused and unfused liners because 0 is in the interval.
c. The null hypothesis is true.
d. There is not enough evidence to suggest mean tensile strength in fused liners is greater than unfused liners because 0 is in the interval.
e. There is enough evidence to suggest mean tensile strength in fused liners is greater than unfused liners because 0 is in the interval.

25. We are interested to know if a student's major area (humanities, sciences, business) affects whether they work a job (no job, part-time job, full-time job). To test this idea, we should use…

a. Two-sample hypothesis test for $\mu_1 - \mu_2$
b. Two-sample hypothesis test for $p_1 - p_2$
c. Chi-Square Test for Independence
d. Chi-Square Goodness-of-Fit test

Part II: Computational Problems

Questions 26 - 29. A study was designed to study **if home environment affects academic achievement** of 12- year-old students. Because genetic differences may also contribute to academic achievement, the researcher wanted to control for this factor. 10 pairs of identical twins were identified, with one twin placed in a home in which academics were emphasized (**Academic**) and the other twin placed in a home in which academics were not emphasized (**Nonacademic**). The final grades for the 20 students are given here.

Twin	Academic	Nonacademic
1	78	71
2	75	70
3	68	66
4	92	85
5	55	60
6	74	72
7	65	57
8	80	75
9	98	92
10	52	56
Population Mean	$\boldsymbol{\mu_A}$	$\boldsymbol{\mu_{NA}}$

26. What should be the appropriate hypotheses to test in terms of μ_A and μ_{NA}?

$$H_0: \mu_A - \mu_{NA} = 0$$

$$H_A: \mu_A - \mu_{NA} \neq 0$$

27. What test should be used?
    a. The 2-sample z test
    b. The 2-sample t test
    c. The paired-t test

28. What is the degrees of freedom of the test that should be used to examine if the new formula is more effective in preventing sunburn?

    df = number of pairs – 1 = 9

29. If the sample yields a test statistic t^* = -2.28 what is the p-value of the test?

$$p = 2 * P(t_9 < -2.28) \approx 2 * P(t_9 < -2.3) = 2 * 0.023 = 0.046$$

<u>Questions 30 - 32.</u> Two random samples, each consisting of 6 rats, were exposed to different environment. One sample of rats was held in a normal environment at 26°C and the other sample was held in a cold 5°C environment. Blood pressures were measured for rats for both groups and listed in the following table. Do the data provide evidence that rats in a cold environment have a higher mean bold pressure than in a normal environment?

26°C		5°C	
Rat	Blood Pressure	Rat	Blood Pressure
1	169	7	390
2	165	8	355
3	168	9	363
4	158	10	372
5	164	11	371
6	172	12	425
Population Mean	$\boldsymbol{\mu_{26}}$		$\boldsymbol{\mu_5}$

30. What should be the appropriate hypotheses to test in terms of μ_{26} and μ_5?

Do rats in cold (5) have *higher* mean than normal (26)? Is $\mu_5 > \mu_{26}$?

$$H_0: \mu_5 - \mu_{26} \leq 0$$

$$H_A: \mu_5 - \mu_{26} > 0$$

31. What test should be used?
    a. The 2-sample z test
    b. The 2-sample t test
    c. The paired-t test

32. What is the most likely value for the degrees of freedom of the test?
    a. ∞
    b. 5
    c. 6
    d. 10 (since only the temperature differs, not the populations of rats, we can likely assume the variances are equal and use $n_1 + n_2 - 2$)
    e. 12

33. A university administrator asserted that upperclassmen spend more time studying than underclassmen. Test this claim using the following information based on random samples from each group of students. Use a 1% level of significance and the P-value approach. Assume the populations are normal and $\sigma_1^2 \neq \sigma_2^2$.

	Sample size	Sample mean	Sample standard deviation
Upperclassmen	28	15.6	2.9
Underclassmen	35	12.3	4.1

Let μ_1 = mean of upperclassmen and μ_2 = mean of underclassmen

$$H_0: \mu_1 - \mu_2 \leq 0$$
$$H_A: \mu_1 - \mu_2 > 0$$

$$df = 60$$

$$Test\ Statistic = t^* = \frac{(\bar{x} - \bar{y}) - \Delta_0}{\widehat{SE}} = \frac{(3.3) - 0}{\sqrt{\frac{(2.9)^2}{28} + \frac{(4.1)^2}{35}}} = \frac{(3.3) - 0}{0.8835} = 3.74$$

$$P - value = P(t_{60} > t^*) = P(t_{60} > 3.74) \approx 0$$

We reject H_0 at 1% level of significance; we conclude that upperclassmen spend more time, on average, studying than underclassmen.

34. A professor believes that women do better on her exams than men do. A sample of 10 women (m = 10) and 12 men (n = 12) yields $\bar{X} = 7$, $\bar{Y} = 5.5$, $S_1^2 = 1$, $S_2^2 = 1.7$. Compute the 99% confidence interval for $\mu_1 - \mu_2$. Assume the populations are normal and that $\sigma_1^2 = \sigma_2^2$. **Interpret your interval in the context of the problem.**

Confidence level = 1 – α = 0.99 => α = 0.01

$$Since\ \sigma_1^2 = \sigma_2^2, use\ S_p^2 = \frac{(m-1)S_1^2 + (n-1)S_2^2}{m+n-2} = \frac{9 + 18.7}{20} = 1.385$$

Degrees of freedom $= n_1 + n_2 - 2 = 20$

$$CV = t_{\frac{\alpha}{2}, df} = t_{0.005,20} = 2.85$$

$$MOE = 2.85 * \sqrt{1.385\left(\frac{1}{10} + \frac{1}{12}\right)} = (2.85) * (0.5039) = 1.436$$

$$1.5 - 1.436 \leq \mu_1 - \mu_2 \leq 1.5 + 1.436$$

$$0.064 \leq \mu_1 - \mu_2 \leq 2.936$$

We are 99% confident that the difference in mean female and male scores falls between 0.064 and 2.936. *(i.e. We would likely find that women do score higher on exams, since 0 is not in the interval.)*

35. Two different makes of stopwatches were used to time 12 different runners over a particular course. Using the information provided below to test H_0: $\mu_d = 0$ vs Ha: $\mu_d \neq 0$ using the rejection region approach. Use $\alpha = 0.01$ and assume the differences are normally distributed.

Runner	1	2	3	4	5	6	7	8	9	10	11	12
Type 1	59	49	64	60	54	47	49	58	66	76	70	66
Type 2	57	46	63	60	50	48	54	54	60	72	72	66

and $\bar{d} = 1.333$, $S_d = 3.114$ where *d* = type 1 - type 2.

$\bar{d} = 1.333, \quad S_d = 3.114 \quad and \ n = 12$

Test Statistic = $t^* = \frac{\bar{d}-\Delta_0}{SE} = \frac{1.333-0}{\frac{S_d}{\sqrt{n}}} = \frac{1.333}{\frac{3.114}{\sqrt{12}}} = \frac{1.333}{0.899} = 1.484$

$CV = t_{\frac{\alpha}{2},df} = t_{0.005,11} = 3.106$

$Since\ it\ is\ a\ two\ sided\ test, we\ have\ rejection\ region: \ \{t^* < -3.106\ or\ t^* > 3.106\}$

Since t^* is not in the RR, we fail to reject H_0 at 1% level of significance.

There is not enough evidence at $\alpha = 0.01$ to support the alternative that a substantial difference exists, on average, in the times taken by stopwatch Type 1 and Type 2.

36. At the 5% significance level, test the claim that there is no preference in the selection of fruit soda flavors for the data observed below.

	Cherry	Strawberry	Orange	Lime	Grape
Observed Frequency	32	28	16	14	10
Expected	$\frac{100}{5} = 20$	**20**	**20**	**20**	**20**
χ^2	$\frac{(32-20)^2}{20} = 7.2$	**3.2**	**0.8**	**1.8**	**5**

$H_0: no\ preference\ in\ flavor\ \left(all\ p_i = {}^1/_5\right)$

$H_A: some\ preference\ in\ flavor$

$\chi^{2*} = 7.2 + 3.2 + 0.8 + 1.8 + 5 = 18$

$RR = \{\chi^{2*} > \chi^2_{4,0.05} = 9.49\}$

Since 18 > 9.49, Reject H_0. There is sufficient evidence at the 5% significance level to suggest a preference of fruit soda flavor exists.

37. A dentist's office is considering mailing reminders for semi-annual teeth cleanings to their entire patient list to see if this increases the number of patients who schedule an appointment within 6 months of their last appointment. Random samples of size m = 35 and n = 40 are selected from their patient list. The first group is mailed a reminder card and the second is not. From those who received reminder cards, 21 scheduled an appointment within 6 months of their last cleaning. Of those who did not receive reminders, 22 scheduled an appointment.
    a. Calculate a 95% confidence interval for the difference in the proportion of patients who schedule a cleaning within 6 months with and without a reminder.
    b. Can we conclude the reminder cards made a difference?

Let p_1 = the proportion receiving reminders, p_2 = the proportion that did not receive reminders

$n_1 = 35,\ \hat{p}_1 = \frac{21}{35} = 0.60$, **and** $n_2 = 40,\ \hat{p}_2 = \frac{22}{40} = 0.55$, **and** $n_1\hat{p}_1, n_1\hat{q}_1, n_2\hat{p}_2, n_2\hat{q}_2 \geq 10$

$z_{\frac{\alpha}{2}} = z_{0.025} = 1.96$

$$(0.60 - 0.55) \pm 1.96\sqrt{\frac{0.60 * 0.40}{35} + \frac{0.55 * 0.45}{40}}$$

0.05 ± 0.2239 = $(-0.1739, 0.2739)$

We are 95% confident that the difference in the proportions of patients who schedule a cleaning within 6 months with and without a reminder is between -0.1739 and 0.2739. Since 0 is in this interval, we cannot conclude that these proportions are significantly different, and so the reminder cards likely did not make a difference.

38. The concentration of benzene was measured in units of milligrams per liter for simple random samples of specimens of wastewater produced at a gas field. The summary statistics are given below.

	Untreated wastewater	Treated wastewater
n	5	7
$\bar{x}$	7.8	3.2
s	1.4	1.7

Satterthwaite df = 9.694. Can you conclude that the mean benzene concentration is less in treated water than in untreated water? Use a 5% significance level.

Let μ_1 = mean of untreated and μ_2 = mean of treated

$H_0 :\ \mu_1 - \mu_2 \leq 0$ **vs** $H_A :\ \mu_1 - \mu_2 > 0$

$$Test\ Statistic = t^* = \frac{(\bar{x} - \bar{y}) - \Delta_0}{\widehat{SE}} = \frac{(7.8 - 3.2) - 0}{\sqrt{\frac{(1.4)^2}{5} + \frac{(1.7)^2}{7}}} = \frac{(4.6) - 0}{0.89714} = 5.13$$

$P - value = P(t_9 > t^*) = P(t_9 > 5.13) \approx 0$

We reject H_0 at 5% level of significance; we have enough evidence to conclude that the mean benzene concentration is less in treated water than in untreated water.

39. A researcher wishes to see if there is a relationship between the hospital and the number of patient infections. A random sample of 3 hospitals was selected, and the number of infections for a specific year has been reported. The data are shown below.

Hospital	Surgical Site Infections	Pneumonia Infections	Bloodstream Infections	Total
A	41	27	51	119
B	36	3	40	79
C	169	106	109	384
Total	246	136	200	582

At α = 0.025, can it be concluded that the number of infections is independent of the hospital in which they occurred?

$\boldsymbol{H_0: Number\ of\ infections\ is\ independent\ of\ hospital}$

$\boldsymbol{H_A: Number\ of\ infections\ depends\ on\ hospital}$

Expected cell counts(left) & Chi-square values (right)

Hospital	**Surgical Site Infections**	**Pneumonia Infections**	**Bloodstream Infections**	**Surgical Site Infections**	**Pneumonia Infections**	**Bloodstream Infections**
A	**50.299**	**27.808**	**40.893**	**1.719**	**0.023**	**2.498**
B	**33.392**	**18.460**	**27.148**	**0.204**	**12.948**	**6.084**
C	**162.309**	**89.732**	**131.959**	**0.276**	**2.949**	**3.994**

$\boldsymbol{\chi^{2*} = 1.719 + 0.023 + \cdots + 2.949 + 3.994 = 30.696}$**, with df =(3-1)(3-1)=4**

$\boldsymbol{RR = \{\chi^{2*} > \chi^2_{4,0.025} = 11.14\}}$

Since the test statistic is in the RR, we Reject H_0. There is sufficient evidence at the 0.025 level to suggest that the number of infections depends on the hospital.

40. In a random sample of 115 American adults who did attend college, 45 said they believe in extraterrestrials. In a random sample of 110 American adults who did not attend college, 39 said they believe. Does this indicate that a difference exists between p_1 (the proportion of people who attended college and believe in extraterrestrials) and p_2 (the proportion who did not attend college and believe)? Use $\alpha = 0.05$.

$\boldsymbol{H_0: p_1 - p_2 = 0}$ **vs** $\boldsymbol{H_A: p_1 - p_2 \neq 0}$

$\boldsymbol{\alpha = 0.05}$

$\boldsymbol{n_1 = 115, x = 45, \hat{p}_1 = \frac{45}{115} = 0.3913, n_2 = 110, y = 39, \hat{p}_2 = \frac{39}{110} = 0.3545,}$

All conditions satisfied.

$$\boldsymbol{\hat{p} = \frac{45 + 39}{115 + 110} = 0.3733, \hat{q} = 0.6267}$$

$$\boldsymbol{z^* = \frac{0.3913 - 0.3455 - 0}{\sqrt{0.3733 * 0.6267\left(\frac{1}{115} + \frac{1}{110}\right)}} = 0.56984}$$

$$\boldsymbol{p = 2 * P(Z > 0.57) = 0.5687}$$

Since p-value is greater than alpha, Fail to Reject H0.
There is not enough evidence at the 5% significance level to suggest that the proportion of people who attended college and believe in extraterrestrials is greater than the proportion of those who did not attend college and believe.

41. Many people take ginkgo supplements advertised to improve memory. Are these over-the- counter ginkgo supplements effective? In a study, 30 elderly adults were assigned to the treatment group(i.e., taking 40mg of ginkgo 3 times a day) or control group (i.e., taking placebo pill 3 times a day). After 6 weeks, the Wechsler Memory Scale was administered. Higher scores indicate better memory function. Summary values are given in the following table:

Group Label	*Group*	*Sample size*	*Sample mean*	*Sample SD*	***Population Mean***
1	Ginkgo	14	5.7	0.6	$\boldsymbol{\mu_1}$
2	Placebo	16	5.5	0.5	$\boldsymbol{\mu_2}$

Based on these results, is there evidence that taking 40mg of ginkgo 3 times a day is effective in increasing mean performance on the Memory Scale? Assume that the population variance of the two groups are **equal.**

a. Perform a test and control the chance of making a type I error at 0.01. (Find both the p-value and rejection region)

b. Report the 99% confidence interval for $\mu_1 - \mu_2$.

$\boldsymbol{H_0 : \mu_1 - \mu_2 \leq 0}$ **vs** $\boldsymbol{H_A : \mu_1 - \mu_2 > 0}$

Since variances are equal, use $s_p^2 = \frac{(14-1)(0.6)^2+(16-1)(0.5)^{\wedge}2}{(14-1)+(16-1)} = 0.301$

$\boldsymbol{Test\ Statistic = t^* = \frac{(\bar{x}-\bar{y})-\Delta_0}{SE} = \frac{(5.7-5.5)-0}{\sqrt{(0.301)*\left(\frac{1}{14}+\frac{1}{16}\right)}} = \frac{0.2}{0.2} = 1}$**, with df = 28**

$\boldsymbol{P-value = P(\ t_{28} > \ t^*) = P(\ t_{28} > 1) = 0.163}$

$\boldsymbol{RR = \{t^* > t_{28,0.01} = 2.467\}}$

We Fail to Reject H_0 at 1% level of significance; we do not have enough evidence to suggest that Gingko significantly increases memory.

$$\boldsymbol{99\%\ CI\ for\ (\mu_1 - \mu_2): (\bar{x}_1 - \bar{x}_2) \pm |t_{0.005,28}| \sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$\boldsymbol{= (5.7 - 5.5) \pm 2.763\sqrt{(0.301) * \left(\frac{1}{14} + \frac{1}{16}\right)}}$$

$$\boldsymbol{= 0.2 \pm 0.553 = (-0.353, 0.753)}$$