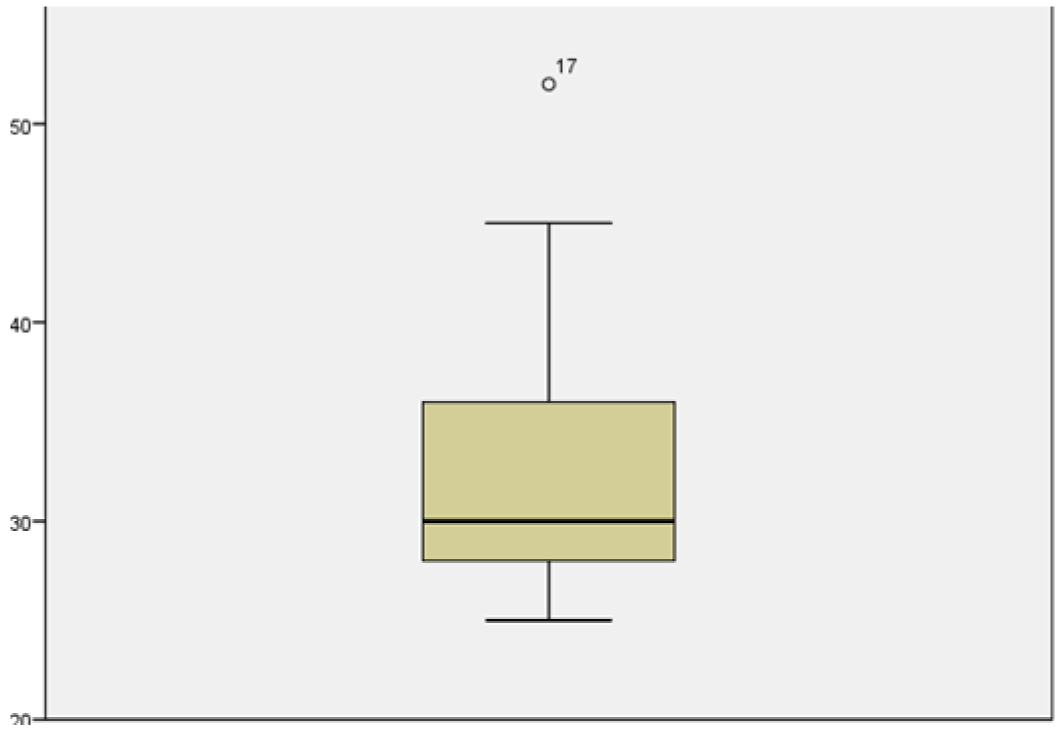
ST370 Exam 1 Review Key

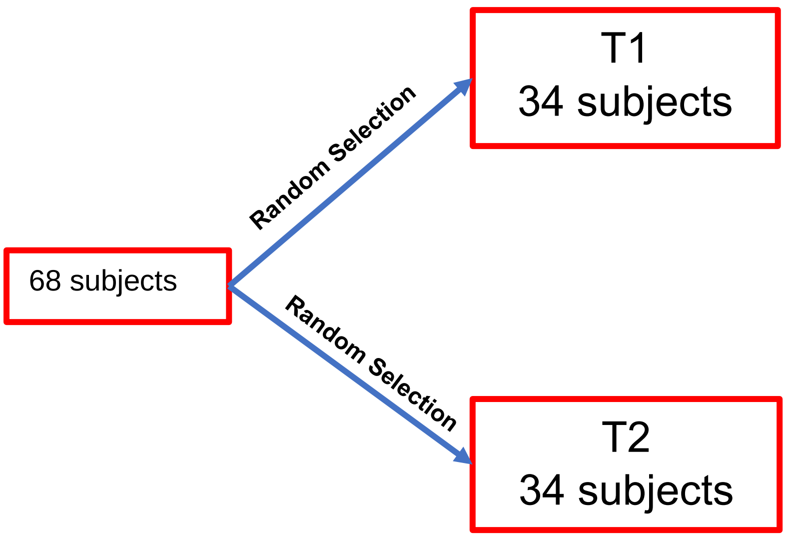
1)
  a. C
  b. A
  c. B
2)
  a. 2 | 5 7 8 8 8 9 9 9
     3 | 0 1 2 4 6 9
     4 | 2 5
     5 | 2
  b. Skewed right and unimodal
  c. 25, 28, 30, 37.5, 52 (btw, mean and standard deviation are irrelevant info)
  d. Yes, 52 is an outlier. Because it is more than Q3+1.5*IQR = 37.5+1.5*(37.5- 28) = 51.75
  e.



  f. Median and IQR since they are resistant to outliers and work best with skewed distributions.
3)
  a. Sports cars (smallest time needed to get to speed)
  b. Small cars
4) Mean = np = 90*0.5 = 4.5; standard deviation = $\sqrt{npq} = \sqrt{90 * 0.5 * 0.95} = 2.068$
5)
  a. A sophomore from Broughton High School.
  b. All 800 sophomores at Broughton High School.
  c. The 150 selected sophomores from Broughton High School.

   d. GPA (quantitative), whether the student took the SAT as a sophomore (categorical).
   e. SRS

6) C
7) E
8)
   a. All preschool children
   b. The 68 preschool children
   c. The amount of improvement in spatial-temporal reasoning (Integers ranging from -4 to 9). Quantitative
   d. Experiment - a treatment (6 months of piano lessons or 6 months of computer lessons) was imposed on the subjects.
   e. Completely randomized design



9)
   a. i
   b. iii

10) C
11) A
12)
   a. $\mu = 0(0.4) + 1(0.1) + 2(0.1) + 3(0.2) + 4(0.1) + 5(0.1) = 1.8$
   b. $\sigma^2 = [0^2*(0.4) + 1^2*(0.1) + 2^2*(0.1) + 3^2*(0.2) + 4^2*(0.1) + 5^2*(0.1)] – (1.8)^2 = 3.16$

13) B. Step 2 is reversed. Plot the z-scores on the x-axis and data on the y-axis.
14)
   a. z~2 :(12.1 to 20.5).
   b. Almost zero: z = (14-16.3)/(2.1/sqrt(25)) = -5.48, P   0
   c. 3.92%: z = (20-16.3)/2.1 = +1.76 P = 1 - .9608 = .0392
   d. 18.988 pounds: z for P   0.9 = 1.28 X = 1.28*2.1 + 16.3

15)
   a. P(F) = 0.35, P(Sp) = 0.28, P(J) = 0.23, P(Sr) = 0.14
      P(C | F) = 0.46. P(C | Sp) = 0.23, P(C | J) = 0.17, P(C | Sr) = 0.14
      P(C) = by Law of Total Probability

= P(C | F)*P(F) + P(C | Sp)*P(Sp) + P(C | J)*P(J) + P(C | Sr)*P(Sr)
=0.35*0.46 + 0.28*0.23 + 0.23*0.17 + 0.14*0.14

b. P(Sr | C) (by Bayes Rule) = $\frac{P(Sr|C)}{P(C)} = \frac{P(C|Sr)*P(Sr)}{P(C)} = \frac{0.14*0.14}{0.2841} = 0.0690$

16)

a. μ_L = 120, σ_L = 7; μ_G = 135, σ_G = 4
X = total worth in dollars = 6L + 8G
Then $\mu_X = \mu_{6L+8G}$ = 6(120) + 8(135) = $1800

b. $\sigma_X^2 = \sigma_{6L+8G}^2 = 6^2\sigma_L^2 + 8^2\sigma_G^2 = 36(7^2) + 64(4^2) = 2788$
Then $\sigma_X = \sqrt{\sigma_X^2} = \sqrt{2788} = \$52.80$

17) 0.8

18) P(A∩B) = 0.1, P(B|A) = 0.5

19)

a. 11.51%: z = (1200 - 1500)/250 = -1.20 P (z<-1.20) = .1151
b. 1887.5 hours: z (for P 0.94) = 1.55 (or 1.56) X = 1.55*250 + 1500 = 1887.5 (or 1890 if z=1.56))
c. 0.9948: z = (1400 -1500)/(250/sqrt(49)) = -2.8 P = .0026 1 - 2P = .9948

20) D

21) E

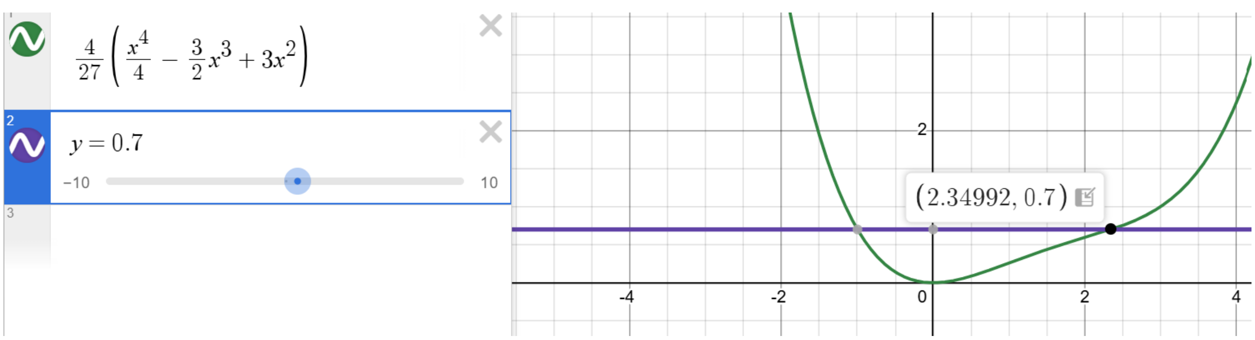
22) C

23) F

24) A

25)

a. $\int_{1.5}^{3} \frac{4}{27} x \left(x^2 - \frac{9}{2}x + 6\right) dx = 0.5625$
b. $0.7 = \frac{4}{27}(\frac{x^4}{4} - \frac{3}{2}x^3 + 3x^2)$ and then use technology to find the value



26)

a. 0.47
b. 0.21
c. 0.46
d. 0.99
e. 0.33
f. 0.01

27) To prove independence, we must show P(A|B) = P(A). Since $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{3/10}{5/10} = \frac{3}{5}$, and P(A) = 6/10 = 3/5, this shows that A and B are independent events.

28)

a. P(x=3) = $\frac{e^{-5.8} \cdot 5.8^3}{3!}$ =0.09845

b. C

29) Let event A=probability of brand A, event R=probability of repair

a. Let P(A) = 0.6, P(A') = 0.4, P(R|A) = 0.25, P(R|A') = 0.10. We want P(A ∩ R) "and". $P(A \cap R) = P(A) \cdot P(R|A) = 0.6 \cdot 0.25 = 0.15$

b. Since we need P(A|R) which is the reverse of what we were given in the problem, we use Bayes. $P(A|R) = \frac{P(R|A) \cdot P(A)}{P(R|A) \cdot P(A) + P(R|A') \cdot P(A')} = \frac{0.25 \cdot 0.6}{0.25 \cdot 0.6 + 0.10 \cdot 0.4} = 0.7895$. Since P(A|R) and P(B|R) are complements (if a TV needs repair, it MUST be either Brand A or Brand B), P(B|R) = 1 – 0.7895 = 0.2105

30)

a. P(x=5) = $\frac{e^{-3} \cdot 3^5}{5!}$=0.1008

b. D

31) C

32) E

33) B

34) E

35) C

36) F

37) Geometric distribution: $= 0.95^7 * 0.05$ = 0.0349, E(X) = 1/p = 1/0.05 = 20

38) B: z = (205 - 202)/4 = .75 P = 1 - .7734 = .2266

39) B

40) C

41)

a. $P(S) = P(S \cap E) + P(S \cap E')$ ➔ 0.46 = 0.27 + $P(S \cap E')$ ➔ $P(S \cap E') = 0.19$

b. $P(E) = P(S \cap E) + P(S' \cap E)$ ➔ 0.40 = 0.27 + $P(S' \cap E)$ ➔ $P(S' \cap E) = 0.13$

c. $P(S \cup E) = P(S) + P(E) - P(S \cap E) = 0.46 + 0.40 - 0.27 = 0.59$
$P(S' \cap E') = P(S \cup E)' = 1 - P(S \cup E) = 1 - 0.59 = 0.41$

42) C

43) F

44) G

45) H

46)

a. P(X=25) = nCr(50,25)*0.6^{25}*0.4^{25} (the solution is 0.4046)

b. np = 50*0.6 = 30, $\sqrt{npq} = \sqrt{50 * 0.6 * 0.4} = 3.464$

47) 11, 23, 32.5, 40, 50

48) 1 | 1
2 | 2 3
3 | 1 2 3 5
4 | 0 1
5 | 0

49) A

50) IQR = 40 – 23 = 17

51) A

52) B

53) P(X > 3.54)
54) P(Z > 2.00) = 0.025.
55)
   a. 3.5
   b. 0.975
   c. 0.025
56) B
57) A
58) $P(\bar{X} > 3.52)$
59) Z = (3.52-3.5)/(0.02/sqrt(20)) = 4.47; P = 1 – 1 ≈ 0
60) A
61) A
62) C
63) G
64) B
65) Lurking variable (for example): Lifestyle
66) 22, 41, 47, 54, 72
67) 2 | 2
    3 | 6
    4 | 0 2 3 7 7 8
    5 | 1 3 5
    6 | 0
    7 | 2
68) Histogram or boxplot – both show a distribution (center and spread)
69) IQR=13 Q1 - 1.5IQR = 21.5 No low outliers Q3 + 1.5IQR = 73.5 No high outliers
70) If at least one sample is positive, the pooled sample would be positive. So, we need probability of no positives and solve using the complement. 1 - P(X=0) = 1 - nCr(8,0)*0.05^0*0.95^8 = 0.3366
71) B
72) P(X<x) = 0.09 = P(Z<z) z = -1.34; x = 63.96
73) P(X > 85) ➔ Z = (85 – 72)/6 ➔ P(Z > 2.17) = 0.015
74) Mean=72 at center. 85 to the right of the mean at the tail. Area to the right of 85 corresponds to P(X>85).
75) $P(\bar{X} < 68)$ ➔ Z = (68 – 72)/(6/sqrt(5) = -1.49 ➔ P(Z < -1.49) = 0.0681
76) A
77) B
78) B
79) E
80) B
81) A
82) B
83) Brown: (36,000-28,000)/5,500=1.45 Andrews:(35,000-30,000)/4,000=1.25 Browns make more.
84) B
85) B
86) C
87) D

88) A
89) F
90) P(X > 700) ➔ Z = (700 – 667)/65 = 0.5077 ➔ P(Z > 0.5077) = 0.3050
91) Mean 667 at center, 700 to the right, area to the right of 700 is P(X > 700).
92) $P(\bar{X} < 655)$ ➔ Z = (655 – 667)/(65/sqrt(10) = -0.58 ➔ P(Z < -0.58) = 0.2810
93) P(X>x) = 0.1 = P(Z>z) ➔ z=1.28, x=750.2
94)

a. $\mu_Z = 25 - 12(20) = -215,\ \sigma_Z^2 = (-12)^2 \cdot 2^2 = 576,\ \sigma_Z = \sqrt{576} = 24$
b. $\mu_Z = 252,\ \sigma_Z^2 = 676,\ \sigma_Z = 26$
c. $\mu_Z = 60,\ \sigma_Z^2 = 53,\ \sigma_Z = \sqrt{53}$
d. $\mu_Z = -20,\ \sigma_Z^2 = 53,\ \sigma_Z = \sqrt{53}$
e. $\mu_Z = 60,\ \sigma_Z^2 = 477,\ \sigma_Z = \sqrt{477}$

95)

a. P(Occasional | Nonsmoker) = 36/155
b. P(Smoker U Occasional) = P(Smoker) + P(Occasional) – P(Smoker ∩ Occasional) = 107/262 + 70/262 – 34/262 = 143/262
c. Show either: P(Smoker) ≠ P(Smoker|Regular) OR
Show that: P(Regular) ≠ P(Regular|Smoker)
P(Smoker) ≠ P(Smoker|Regular)
107/262 ≠ 61/139 ➔ 0.4084 ≠ 0.4388

96) C ➔ P(at least one) = 1 – P(0) = 1 – 0.8^3
97) Find blank cell first… ΣP(x) = 1 ➔ P(2) = 1 – 0.1 – 0.3 – 0.1 = 0.5

a. E(X) = (-1)*0.1 + (0)*0.3 + (1)*0.3 + (2)*0.5 = 1
b. (using shortcut method) ➔ V(X) = $[(-1)^2(0.1) + (0)^2*0.3 + (1)^2*0.1 + (2)^2*0.5] - 1^2$ = 1.2 ➔ standard deviation is square root of V(X) = $\sqrt{1.2} = 1.095$

98) P(X < 15)
99) Z = (15 – 30)/5 = -3.00
100) 0.0015
101)

a. 0
b. -3.00
c. 0.0015
d. 0.9985

102)

x	1	2	3	4	5	6
$P(X=x)$	0.06	0.06	0.12	0.20	0.41	0.15

a. 0
b. 0.06 + 0.06 = 0.12
c. 0.41 + 0.15 = 0.56

103)

a. P(X ≥ 0.40) = 0.60
b. P(X = 0.40) = 0
c. P(0.40 < X < 1.40) = 0.60.
d. P(0.22 ≤ X ≤ 0.25 or 0.42 ≤ X ≤ 0.45) = 0.06.
e. 0.7

104) $P(B|A) = \frac{0.18*0.52}{0.18*0.52+0.57*0.48} = 0.255$

105)

a. A and B cannot both occur. $P(A \cap B) = 0$. P(A or B) = P(A) + P(B) = 0.2 + 0.6 = 0.8

b. S is the sample space. P(S) = 1 means that some event in the sample space must occur.

c. The event is called A complement. P(A') = 1 – P(A) = 1 – 0.7 = 0.3.

d. The event {A and B} is the outcome that both A and B occur simultaneously. For independent events, the multiplication rule says $P(A \cap B) = P(A) \cdot P(B) = (0.8)(0.3) = 0.24$

e. No, since disjoint means $P(A \cap B) = 0$, this would imply $P(A \cup B) = P(A) + P(B)$ but the sum would exceed 1 and probability must be between 0 and 1.

106)

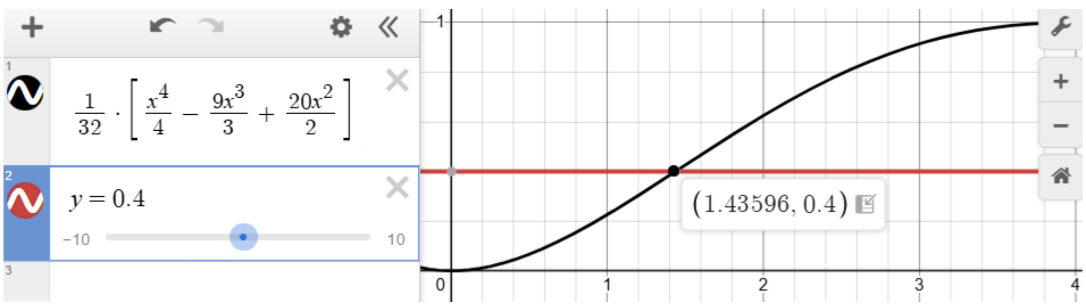
a. $\int_1^3 \frac{1}{32}(x^3 - 9x^2 + 20x)\, dx = \frac{11}{16}$

b. $E(X) = \int_0^4 x\left[\frac{1}{32}(x^3 - 9x^2 + 20x)\right] dx = \frac{26}{15} \approx 1.733$

c. $V(X) = \int_0^4 x^2\left[\frac{1}{32}(x^3 - 9x^2 + 20x)\right] dx - E(X)^2 = \frac{56}{15} - \left(\frac{26}{15}\right)^2 = \frac{164}{225} \approx 0.7289$

➔ standard deviation = $\sqrt{\frac{164}{225}} \approx 0.8537$

d. 1.43596



107) C ➔ P(at least one W) = 1 – P(no W's) = 1 – $(25/26)^4$

108) This problem can be viewed as a uniform distribution or using calculus.

a. P(0 ≤ X ≤ 2) = 2*(1/8) = ¼

b. P(X = 6) = 0 ➔ X is a continuous variable!

c. $\int_0^8 x \cdot f(x)\, dx = \int_0^8 \frac{1}{8} x\, dx = \frac{x^2}{16}\Big]_0^8 = \frac{64}{16} = 4$