Lecture 19

One-Way ANOVA

Today's Updates / Reminders

- PRWA (Article) submission is due at the end of the day TODAY.
- No lab or HW due this week.
- Homework 9 is open and due in a week and a half.
- Lab 8 – MATLAB – One-Way ANOVA opens Friday
- Exams have been published. You have one week to submit regrade requests.

Quick Review

ANOVA It is a method for comparing several (two or more) population means.

We draw a SRS from each population and use the data to test the null hypothesis that the population means are all equal.

Hypothesis→

$$H_0 : \mu_1 = \mu_2 = ...\mu_I$$

$$H_a : \text{not all the } \mu_i \text{ are equal}$$

NOTE: this H_a is NOT the same thing as saying "all the means are different."

Assumptions:

Need independent SRSs from each population.

Populations assumed to be Normal with the same standard deviation (same width or spread). → **Need to check**

largest sample std < 2 (smallest sample std), then it is ok to pool variances and run ANOVA.

Test statistic for ANOVA

F test statistic → F distribution

$$\text{F test statistic} = \frac{\text{variation among the means of the groups}}{\text{variation within the groups}}$$

Characteristics of the F Statistic:

- The F values are positive
- Its distribution (F distribution) is non-symmetric (skewed to the right)
- Mean of the distribution approximately 1.
- F >>1 comes together with small P-val. → Evidence against null hypothesis.

Example One-way ANOVA

Suppose the USGA wants to compare mean distances associated with four different brands of golf balls when struck with a driver. The USGA's robotic golfer, Iron Byron, uses a driver to hit a random sample of 10 balls of each brand in a random sequence. The distance is recorded for each hit and then results are shown below by brand:

Treatment	Distance (yards)									
Brand A	254.2	248.7	251.3	254.4	264.9	253.6	257.5	248.1	258.0	252.8
Brand B	263.2	262.9	265.0	254.5	264.3	257.0	262.8	264.4	260.6	255.9
Brand C	269.7	263.2	277.5	267.4	270.5	265.5	270.7	272.9	275.6	266.5
Brand D	251.6	248.6	249.4	242.0	246.5	251.3	261.8	249.0	247.1	245.9

What is the factor?
Response variable?
Number of groups (*I*)?
Number in each group(ni)?
Total number of measurements?

Right click this link and choose Copy Link, and then in StatCrunch, choose Data, Load, From File, On the Web. Paste the link and change Delimiter to Comma
GolfBalls.csv

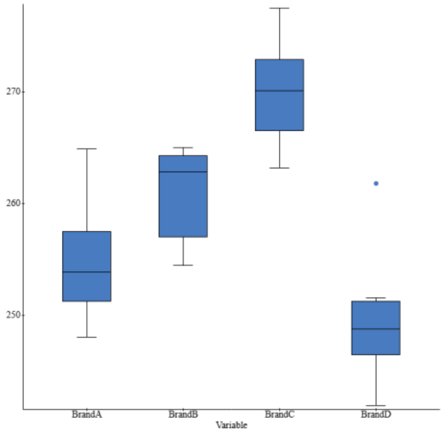
1. Use graphical and numerical methods to describe the data:

Stat, Summary Stats, Columns

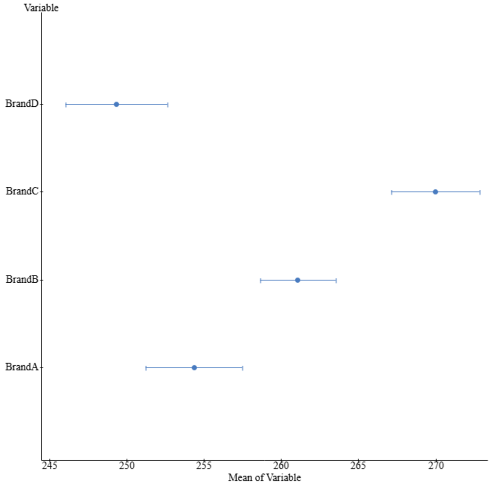
Summary statistics:

Column	n	Mean	Variance	Std. dev.	Std. err.	Median	Range	Min	Max	Q1	Q3
BrandA	10	254.35	24.202778	4.9196319	1.5557242	253.9	16.8	248.1	264.9	251.3	257.5
BrandB	10	261.06	14.947111	3.8661494	1.2225838	262.85	10.5	254.5	265	257	264.3
BrandC	10	269.95	20.258333	4.5009258	1.4233177	270.1	14.3	263.2	277.5	266.5	272.9
BrandD	10	249.32	27.072889	5.2031614	1.6453841	248.8	19.8	242	261.8	246.5	251.3

Graph, Boxplot



Graph, Means Plot



Stat, ANOVA, One Way

ANOVA in StatCrunch

- Select all 4 columns, click Compute
- H_0: $\mu_{BrandA} = \mu_{BrandB} = \mu_{BrandC} = \mu_{BrandD}$
- H_a: not all of the μ are the same (at least one is different)
- F = 36.85, p-value = <0.0001 < 0.05
- At 5% significance, reject H_0.
- We have evidence that not all the population distances are the same.
- S_p = pooled estimate for the standard deviation = $\sqrt{MSE} = \sqrt{21.620278} = 4.6498$

We analyzed std dev already!

Analysis of Variance results:
Data stored in separate columns.

Column statistics

Column	n	Mean	Std. Dev.	Std. Error
BrandA	10	254.35	4.9196319	1.5557242
BrandB	10	261.06	3.8661494	1.2225838
BrandC	10	269.95	4.5009258	1.4233177
BrandD	10	249.32	5.2031614	1.6453841

ANOVA table

Source	DF	SS	MS	F-Stat	P-value
Columns	3	2390.354	796.78467	36.853581	<0.0001
Error	36	778.33	21.620278		
Total	39	3168.684			

Coefficient of Determination:

It is an additional parameter given as output by some statistical packages:

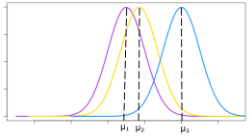
$$R^2 = \frac{SSG}{SST}$$

It represents the percentage of variation in the data that is explained by the ANOVA model.
It estimates the standard error of the model, i.e. the error we produce when we pool the variances.
In this problem:

ANOVA table

Source	DF	SS	MS	F-Stat	P-value
Columns	3	2390.354	796.78467	36.853581	<0.0001
Error	36	778.33	21.620278		
Total	39	3168.684			

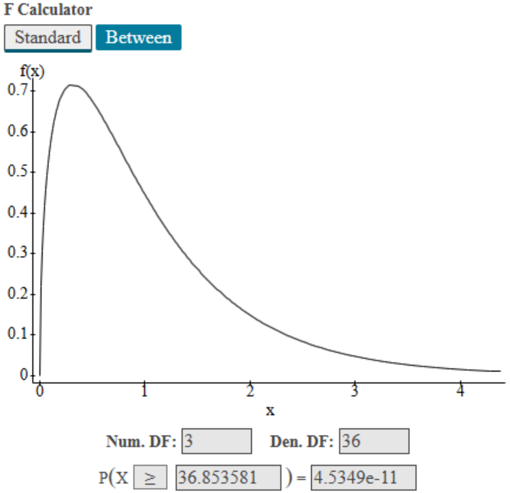
Output Calculations



- R^2 =
- Pooled standard deviation
- MSE =
- F-Stat =
- P-value =

ANOVA table

Source	DF	SS	MS	F-Stat	P-value
Columns	3	2390.354	796.78467	36.853581	<0.0001
Error	36	778.33	21.620278		
Total	39	3168.684			



SALT Output

To read this data for SALT, we must use column-based data, not a “spreadsheet” table.

Settings

Procedure Selection: ANOVA

Dataset Contains: ( ) Summary Statistics (•) Raw Data

Responses: DISTANCE

Factors: BRAND

Confidence Interval: 95 %

Generate Results | Reset

SUMMARY DATA | POST HOC ANALYSIS | SIDE-BY-SIDE BOXPLOTS

Null Hypothesis **H_0: All group means are equal**
Alternative Hypothesis **H_1: Not all group means are equal**

Summary Statistics by Factor

Factor	n	Mean	Standard Deviation	95% Confidence Interval
BrandA	10	254.35	4.919632	(251.367924 , 257.332076)
BrandB	10	261.06	3.866149	(258.077924 , 264.042076)
BrandC	10	269.95	4.500926	(266.967924 , 272.932076)
BrandD	10	249.32	5.203161	(246.337924 , 252.302076)

Analysis of Variance Table

Source	df	Sum of Squares	Mean Squares	F-Statistic	P-Value
Treatments	3	2390.354	796.7846667	36.8535813	0
Error	36	778.33	21.6202778		
Total	39	3168.684			

Treatments may also be referred to as *Between Groups*. *Error* may also be referred to as *Within Groups*.

BRAND	DISTANCE
BrandA	254.2
BrandA	248.7
BrandA	251.3
BrandA	254.4
BrandA	264.9
BrandA	253.6
BrandA	257.5
BrandA	248.1
BrandA	258
BrandA	252.8
BrandB	263.2
BrandB	262.9
BrandB	265
BrandB	254.5
BrandB	264.3
BrandB	257
BrandB	262.8
BrandB	264.4
BrandB	260.6
BrandB	255.9
BrandC	269.7
BrandC	263.2
BrandC	277.5
BrandC	267.4
BrandC	270.5
BrandC	265.5
BrandC	270.7
BrandC	272.9
BrandC	275.6
BrandC	266.5
BrandD	251.6
BrandD	248.6
BrandD	249.4
BrandD	242
BrandD	246.5
BrandD	251.3
BrandD	261.8
BrandD	249
BrandD	247.1
BrandD	245.9

Filling Out the ANOVA Table

- Using the previous slide, we are going back-fill this table given the two numbers in red. This is homework 9, #6.

An experiment was carried out to compare electrical resistivity for six different low-permeability concrete bridge deck mixtures. There were 26 measurements on concrete cylinders for each mixture; these were obtained 28 days after casting. The entries in the accompanying ANOVA table are based on information in an article. Fill in the remaining entries. (Round your answer for *f* to two decimal places.)

Source	df	Sum of Squares	Mean Square	*f*
Mixture				
Error			13.859	
Total		5664.345		

- Btw, what is s_p? R^2?

Example

Example:

A veterinary epidemiologist wants to see if 3 food supplements result in different mean milk yields in dairy cows. You assign 15 cows, 5 per food supplement. Complete the ANOVA table below and answer the question: At the 0.05 level, is there a difference in mean yields?

Food1	Food2	Food3
25.40	23.40	20.00
26.31	21.80	22.20
24.10	23.50	19.75
23.74	22.75	20.60
25.10	21.60	20.40

Source	Degrees of freedom (df)	Sums of Squares (SS)	Mean Squares (MS)	F Statistic
Groups/ Treatments				
Error		11.0532		
Total		58.2172		

You've Rejected H_0 – now what?

Bonferroni Corrections

Suppose with ANOVA you reject H_0 and …

You have evidence that not all the population means are the same.

But which means are different?
ANOVA won't tell you that!

After rejecting H_0 using ANOVA we need to do something else→ **Bonferroni Multiple Comparisons test** will show the differences.

Bonferroni Multiple Comparisons

- It consists of simultaneous 2-sample comparison of means *t* tests. Each row of the output deals with one pair of means. For each pair:
  (H_0: $\mu_1 - \mu_2 = 0$, H_a: $\mu_1 - \mu_2 \neq 0$, for each 2-sample t test)

- It tells you that there is a significant difference between 2 groups if:
  - P-value $\leq \alpha$
  - CI for that difference does NOT contain 0

(Remember H_a: $\mu_1 - \mu_2 \neq 0$ (two-sided) & $C + \alpha = 100$, then it is OK to use CI for significance test)

Bonferroni tests for our example

$$H_0: \mu_{BrandA} - \mu_{BrandB} = 0$$
$$H_a: \mu_{BrandA} - \mu_{BrandB} \neq 0$$

$$H_0: \mu_{BrandB} - \mu_{BrandC} = 0$$
$$H_a: \mu_{BrandB} - \mu_{BrandC} \neq 0$$

$$H_0: \mu_{BrandC} - \mu_{BrandD} = 0$$
$$H_a: \mu_{BrandC} - \mu_{BrandD} \neq 0$$

$$H_0: \mu_{BrandA} - \mu_{BrandC} = 0$$
$$H_a: \mu_{BrandA} - \mu_{BrandC} \neq 0$$

$$H_0: \mu_{BrandB} - \mu_{BrandD} = 0$$
$$H_a: \mu_{BrandB} - \mu_{BrandD} \neq 0$$

$$H_0: \mu_{BrandA} - \mu_{BrandD} = 0$$
$$H_a: \mu_{BrandA} - \mu_{BrandD} \neq 0$$

How many ways can I compare pairs of 4 items if order doesn't matter?

nCr(4,2) = 6

If there are *k* treatment means, there are:

$$c = \frac{k(k-1)}{2}$$

pairs of means that can be compared

P-value for multiple comparisons

!! If I do all 6 comparisons and use α = 0.05 for each test, I am going to proliferate my error past the original α for the one-way ANOVA !!

Tests	Reject	Fail to Reject
BrandA – Brand B	0.05	0.95
…	0.05	0.95
BrandC – Brand D	0.05	0.95

ANOVA says we will reject at alpha = 0.05 if **<u>at least one</u>** mean is different from one of the others.

What is the opposite of at least one?

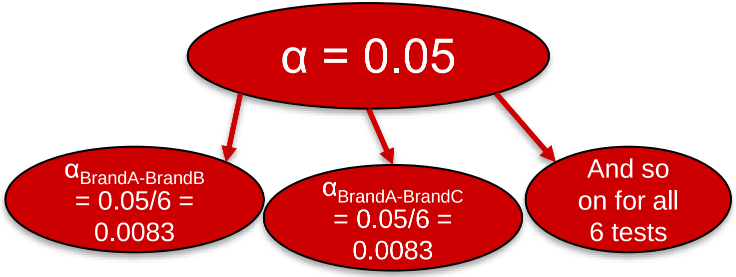
X (# of rejected H_0's)	P(X)
x=0	nCr(4,0)*0.95*0.95*0.95*0.95 = 0.8145
x=1	nCr(4,1)*0.95*0.95*0.95*0.05 = 0.1715
x=2	nCr(4,2)*0.95*0.95*0.05*0.05 = 0.0135
x=3	nCr(4,3)*0.95*0.05*0.05*0.05 = 0.0005
x=4	nCr(4,4)*0.05*0.05*0.05*0.05 = 6.25e-6

$\alpha_E = 1 - (1 - \alpha_I)^c$, where c is the number of tests to be carried out

More on Bonferroni

- The math for Bonferroni is the most simplified and avoids the use of specialized tables. Your text relies on Tukey. Other methods include LSD (least squares difference) and Dunnett.
- I made a few homemade homework questions to demonstrate Bonferroni and any exposition on it will come from my slides.
- Very simply stated, Bonferroni takes the alpha and splits it up among the number of paired tests. It is technically a post-hoc correction scheme to adjust the p-value resulting from the ANOVA.
- So, as we calculate this, we will find a t-test statistic for a pairwise comparison, find its p-value and we will multiply that by the number of tests.

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{MSE\left[\frac{1}{n_1}+\frac{1}{n_2}\right]}}$$



But instead of dividing alpha by the # of tests, we will multiply our p-values by the # of tests.

The downside: Bonferroni causes excessive β

Bonferroni Comparison

- Let's test Brand A vs. Brand D on the golf balls problem. So as stated before, we will use hypotheses:

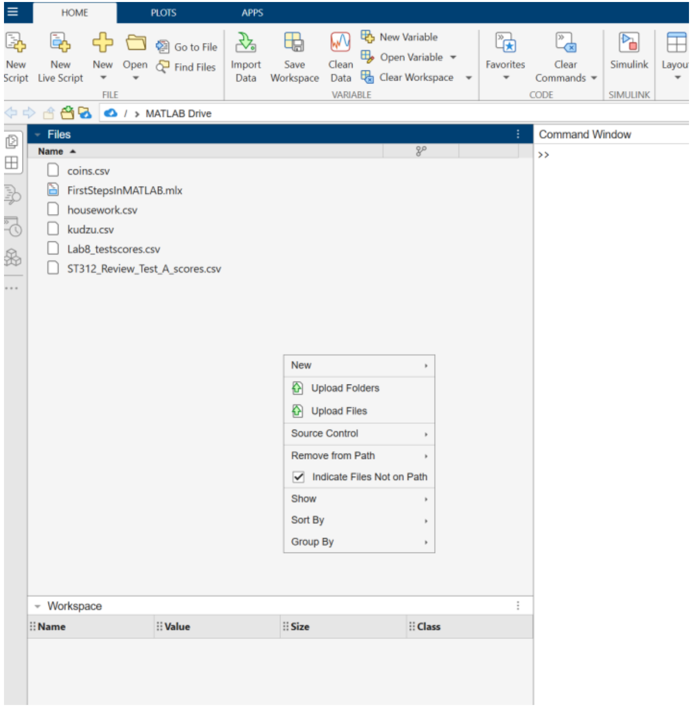
$$H_0: \mu_{BrandA} - \mu_{BrandD} = 0$$
$$H_0: \mu_{BrandA} - \mu_{BrandD} \neq 0$$

- Calculate the test statistic, find df (which is based on DFE), and then find p-value. Multiply a one-sided p-value by 2 for a two-tailed result. Then multiply by the number of possible comparisons.

Technology

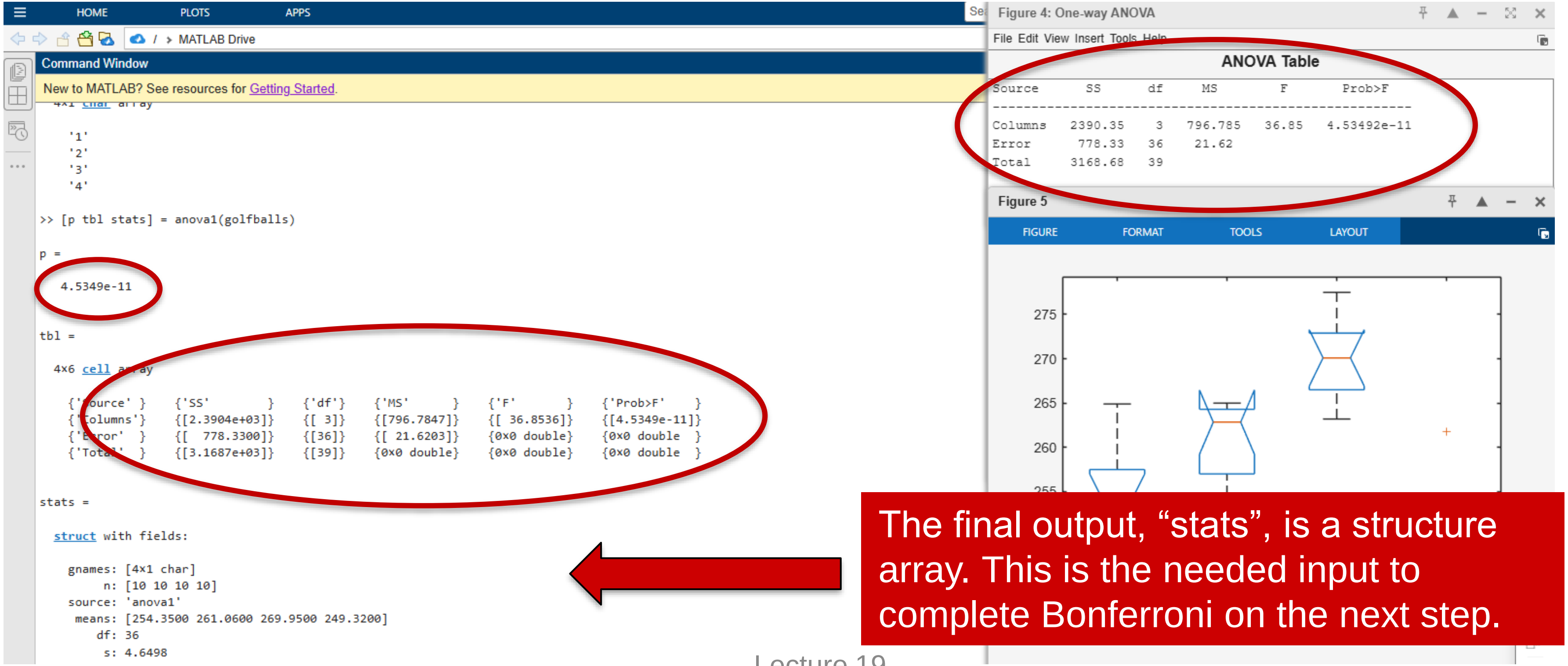
- This is not a process you want to repeat often. Imagine having 5 groups and doing pairwise comparisons… nCr(5,2) = 10 calculations.
- Lab 8 is going to encourage us to use MATLAB.
- Let's do the golf ball example together in MATLAB (since your lab will follow this same process).
- Once you download the csv file to your computer, you need to load it into the MATLAB working directory.
- Right-click in the Files area and choose Upload Files and select the downloaded file.
- Then in the command window, type:

  golfballs = readmatrix('golfballs.csv')



- This command assumes the data are grouped in columns (and they are). When you put variables on the left side of the equals, MATLAB will assign each of them values.

>> [p tbl stats] = anova1(golfballs)



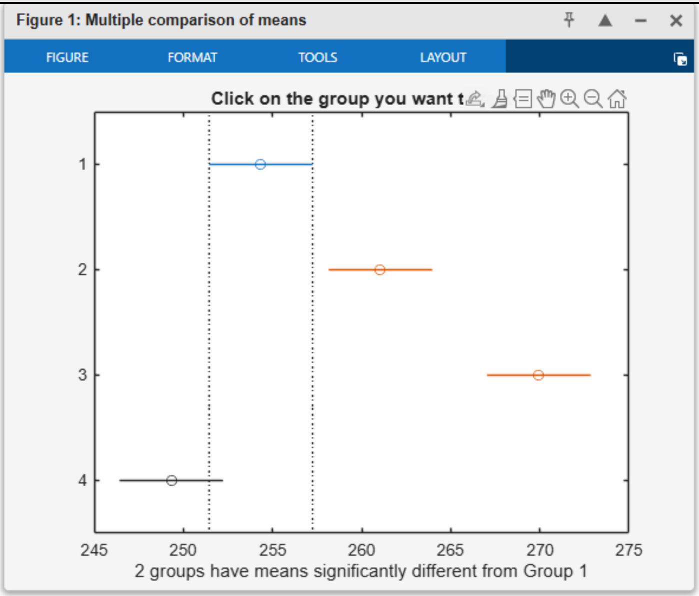
The final output, "stats", is a structure array. This is the needed input to complete Bonferroni on the next step.

- This command will invoke multiple comparisons. The default is Tukey, but there are many options… try erasing Bonferroni and typing quotes…

>> close all #I was having issues with the display of the means plot so **close all** will close other figure windows.

>> [c m h gnames] = multcompare(stats,"CriticalValueType","bonferroni")

```
>> [c m h gnames] = multcompare(stats,"CriticalValueType", "bonferroni")

c =

    1.0000    2.0000  -12.5157   -6.7100   -0.9043    0.0160
    1.0000    3.0000  -21.4057  -15.6000   -9.7943    0.0000
    1.0000    4.0000   -0.7757    5.0300   10.8357    0.1245
    2.0000    3.0000  -14.6957   -8.8900   -3.0843    0.0008
    2.0000    4.0000    5.9343   11.7400   17.5457    0.0000
    3.0000    4.0000   14.8243   20.6300   26.4357    0.0000


m =

  254.3500    1.4704
  261.0600    1.4704
  269.9500    1.4704
  249.3200    1.4704


h =

  Figure (1: Multiple comparison of means) with properties:

      Number: 1
        Name: 'Multiple comparison of means'
       Color: [0.9608 0.9608 0.9608]
    Position: [1628 1512 583 437]
       Units: 'pixels'

  Show all properties
```



1 = Brand A
2 = Brand B
3 = Brand C
4 = Brand D

Bonferroni Output

- Looking more closely at the Bonferroni output…

$$(\bar{x}_i - \bar{x}_j) \pm m$$

c =

Group$_i$	vs Group$_j$	CI Lower Limit	$(\bar{x}_i - \bar{x}_j)$	CI Upper Limit	P-value
1.0000	2.0000	-12.5157	-6.7100	-0.9043	0.0160
1.0000	3.0000	-21.4057	-15.6000	-9.7943	0.0000
1.0000	4.0000	-0.7757	5.0300	10.8357	0.1245
2.0000	3.0000	-14.6957	-8.8900	-3.0843	0.0008
2.0000	4.0000	5.9343	11.7400	17.5457	0.0000
3.0000	4.0000	14.8243	20.6300	26.4357	0.0000

At 5% significance level,

We have evidence in favor of:

$\mu_{BrandA}-\mu_{BrandB}\neq 0$
$\mu_{BrandA}-\mu_{BrandC}\neq 0$

We don't have evidence in favor of $\mu_{BrandA}-\mu_{BrandD}\neq 0$

Continue the process for the remaining comparisons: Brand B is different than C and D and Brand C is different than Brand D

We have evidence the population distance for Brand A (1) is significantly different from Brand B (2) and Brand C (3), but Brand A (1) and Brand D (4) are not significantly different from each other.

Appendix: Tukey's Procedure

- Tukey's procedure involves the use of another probability distribution called the **Studentized range distribution.** The distribution depends on two parameters: a numerator df *m* and a denominator df *v*.
- Let $Q_{\alpha,m,n}$ denote the upper-tail α critical value of the Studentized range distribution with *m* numerator df and *v* denominator df (analogous to F_{α,v_1,v_2}).
- Values of $Q_{\alpha,m,n}$ are given in Appendix Table A.10.
- Since we are not really interested in the lower and upper limits of the various intervals but only in which include 0 and which do not, much of the arithmetic associated with the formula can be avoided.

With probability $1-\alpha$,

$$\bar{X}_{i\cdot} - \bar{X}_{j\cdot} - Q_{\alpha,I,I(J-1)}\sqrt{\text{MSE}/J} \le \mu_i - \mu_j \le \bar{X}_{i\cdot} - \bar{X}_{j\cdot} + Q_{\alpha,I,I(J-1)}\sqrt{\text{MSE}/J} \quad (10.4)$$

for every i and j ($i = 1,\ldots, I$ and $j = 1,\ldots, I$) with $i < j$.

Appendix: Tukey's Procedure (the *T* Method)

- Suppose, for example, that I = 5 and that

$$\bar{x}_2 < \bar{x}_5 < \bar{x}_4 < \bar{x}_1 < \bar{x}_3$$

Then

- **1.** Consider first the smallest mean $\bar{x}_2$. If $\bar{x}_5 - \bar{x}_2 \geq w$, proceed to Step 2. However, if $\bar{x}_5 - \bar{x}_2 < w$, connect these first two means with a line segment.

  Then if possible extend this line segment even further to the right to the largest $\bar{x}_i$, that differs from $\bar{x}_2$. by less than w (so the line may connect two, three, or even more means).

Appendix: Tukey's Procedure (the *T* Method)

- **2.** Now move to $\bar{x}_5$ and again extend a line segment to the largest $\bar{x}_i$ to its right that differs from $\bar{x}_5$, by less than w (it may not be possible to draw this line, or alternatively it may underscore just two means, or three, or even all four remaining means).

  3. Continue by moving to $\bar{x}_4$ and repeating, and then finally move to $\bar{x}_1$.

Appendix: Tukey’s Procedure (the *T* Method)

- To summarize, starting from each mean in the ordered list, a line segment is extended as far to the right as possible as long as the difference between the means is smaller than *w*.

- It is easily verified that a particular interval of the form indicated in the previous formula will contain 0 if and only if the corresponding pair of sample means is underscored by the same line segment.

Golf Balls Using Tukey (SALT and SAS)

Tukey-Kramer Post Hoc Analysis at 95% Confidence Interval

BrandA - BrandB Interval: (-12.310394 , -1.109606)
BrandA - BrandC Interval: (-21.200394 , -9.999606)
BrandA - BrandD Interval: (-0.570394 , 10.630394)
BrandB - BrandC Interval: (-14.490394 , -3.289606)
BrandB - BrandD Interval: (6.139606 , 17.340394)
BrandC - BrandD Interval: (15.029606 , 26.230394)

- Some textbooks/software show a different output style. There will be a bar extending over the group means and any groups that are not dissimilar will have a bar covering both.
- In this case, Brand A and Brand D are both covered by a green bar. The interpretation is that Brand A and Brand D are NOT different.
- The blue bar only covers Brand C which says that Brand C is different from all other groups.
- Sometimes, the means themselves are displayed in a row and the output looks something like:

$$\overline{269.95} \quad \overline{261.06} \quad \overline{254.35 \quad 249.32}$$

where the overline covers the means that are similar.

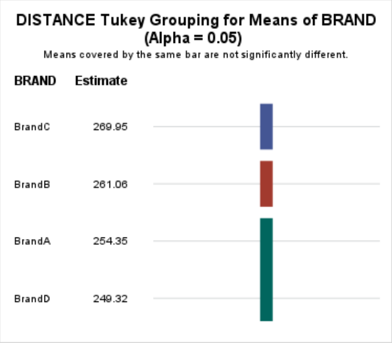
One-Way ANOVA with Tukey post hoc

The ANOVA Procedure

Tukey's Studentized Range (HSD) Test for DISTANCE

Note: This test controls the Type I experimentwise error rate, but it generally has a higher Type II error rate than REGWQ.

Alpha	0.05
Error Degrees of Freedom	36
Error Mean Square	21.62028
Critical Value of Studentized Range	3.80880
Minimum Significant Difference	5.6004



Test at end of Middle School: one-way ANOVA example

A study compared the scores of students from three different school corporations in a test taken at the end of middle school. The results are shown in the table.

Today.uconn.edu

Corp.1	Corp.2	Corp.3
499	490	585
620	395	647
469	402	477
485	177	445
660	475	485
588	617	703
675	616	528
517	587	465
649	528	
209	518	
404	370	
738	431	
628	518	
609	639	
617	368	
704	538	
558	519	
653	506	
548		

School Data

Test at the end of Middle School example

Factor:

Response variable:

I =

n_1 = , n_2 = , n_3 =

N =

Corp.1	Corp.2	Corp.3
499	490	585
620	395	647
469	402	477
485	177	445
660	475	485
588	617	703
675	616	528
517	587	465
649	528	
209	518	
404	370	
738	431	
628	518	
609	639	
617	368	
704	538	
558	519	
653	506	
548		

School Data

Test at the end of Middle School summary statistics

Means and standard deviations for each group

Corp.	Mean	N	Std. Deviation	Median	Minimum	Maximum
1	570.00	19	122.958	609.00	209	738
2	483.00	18	112.948	512.00	177	639
3	541.88	8	93.963	506.50	445	703
Total	530.20	45	118.906	528.00	177	738

Is it appropriate to pool the variances?

Test at end of Middle School: one-way ANOVA

H_0: $\mu_1 = \mu_2 = \mu_3$

H_a: Not all the population means are the same for these groups.

Assume α = 0.05

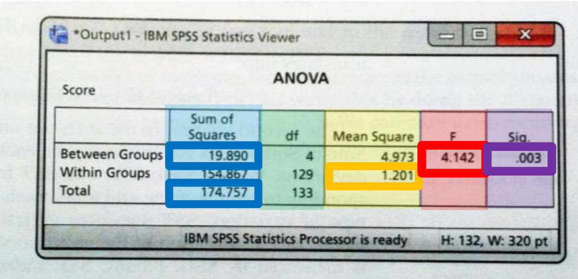
ANOVA

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	71288.325	2	35644.163	2.718	.078
Within Groups	550810.9	42	13114.545		
Total	622099.2	44			

S_p =

R^2 =

After running ANOVA: Interpret the ANOVA table



ANOVA

Score

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	19.890	4	4.973	4.142	.003
Within Groups	154.867	129	1.201		
Total	174.757	133			

Source	Sum of Squares	Degrees of Freedom	Mean Square	F	Sig.
Groups *(Between Groups)*	SSG	DFG = I - 1	$MSG = \frac{SSG}{DFG}$	$\frac{MSG}{MSE}$	*P*-value
Error *(Within Groups)*	SSE	DFE = N - I	$MSE = \frac{SSE}{DFE} = s_p^2$		
Total	SST	DFT = N - 1			

Coefficient of Determination: $R^2 = \frac{SSG}{SST}$ (multiply by 100 to write as percent)

If R^2 is close to 100, most of the error comes from the different groups.

Pooled standard deviation $s_p = \sqrt{MSE}$ $\rightarrow$ it is the estimate for σ (populations std)

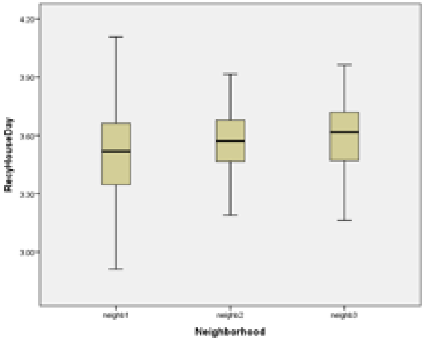
Test-statistic for ANOVA $\rightarrow$ F test-statistic (F distribution is right skewed)

If P-value $\leq \alpha \rightarrow$ reject $H_0 \rightarrow$ perform Bonferroni

If P-value $> \alpha \rightarrow$ do not reject $H_0 \rightarrow$ do not perform Bonferroni

USE THE FOLLOWING TO ANSWER QUESTIONS 16-19 IN THE NEXT PAGE.

Is there evidence that not all population means of pounds of recyclables produced daily per household are the same for different neighborhoods? Below are SPSS outputs obtained from analyzing data collected from three different neighborhoods. (Write the letter corresponding to your answer on the line provided).



Descriptives

RecyHouseDay

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean: Lower Bound	95% Confidence Interval for Mean: Upper Bound	Minimum	Maximum
1.00	79	3.5224	.22367	.02516	3.4723	3.5725	2.91	4.11
2.00	76	3.5611	.16237	.01863	3.5240	3.5982	3.19	3.92
3.00	80	3.5971	.18022	.02015	3.5570	3.6372	3.16	3.96
Total	235	3.5603	.19246	.01255	3.5356	3.5851	2.91	4.11

ANOVA

RecyHouseDay

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	.222	2	.111	3.052	.049
Within Groups	8.445	232	.036		
Total	8.667	234			

Multiple Comparisons

Dependent Variable: RecyHouseDay

Bonferroni

(I) neighb	(J) neighb	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval: Lower Bound	95% Confidence Interval: Upper Bound
1.00	2.00	-.03871	.03066	.624	-.1126	.0352
	3.00	-.07475*	.03026	.043	-.1477	-.0018
2.00	1.00	.03871	.03066	.624	-.0352	.1126
	3.00	-.03604	.03056	.718	-.1097	.0377
3.00	1.00	.07475*	.03026	.043	.0018	.1477
	2.00	.03604	.03056	.718	-.0377	.1097

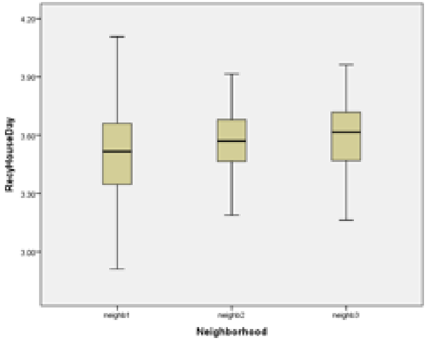
*. The mean difference is significant at the 0.05 level.

________ 16. (3 points) Based on the side-by-side boxplot we might conclude

A. There to be no significant difference between the mean of pounds of daily recyclables per household in neighborhood 1 and neighborhoods 2 and 3 in our samples.
B. There to be a significant difference between the mean of pounds of daily recyclables per household in neighborhood 1 and neighborhoods 2 and 3 in our samples because the medians in the side-by-side boxplot are at different heights.
C. There to be no significant difference between the mean of pounds of daily recyclables per household in neighborhood 1 and neighborhoods 2 and 3 in the populations.
D. There to be a significant difference between the mean of pounds of daily recyclables per household in neighborhood 1 and neighborhoods 2 and 3 in the populations because the medians in the side-by-side boxplot are at different heights..

________ 17. (3 points) Regarding pooling the variances:

A. It is OK to do so because 0.22367<2*0.16237.
B. It is OK to do so because 0.19246<2*0.16237.
C. Variances are not given in the SPSS outputs therefore we cannot answer this question.



Descriptives

RecyHouseDay

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean Lower Bound	95% Confidence Interval for Mean Upper Bound	Minimum	Maximum
1.00	79	3.5224	.22367	.02516	3.4723	3.5725	2.91	4.11
2.00	76	3.5611	.16237	.01863	3.5240	3.5982	3.19	3.92
3.00	80	3.5971	.18022	.02015	3.5570	3.6372	3.16	3.96
Total	235	3.5603	.19246	.01255	3.5356	3.5851	2.91	4.11

_______ 18. (3 points) Based on the ANOVA test we can conclude

A. At the 5% significance level there is evidence that at least one of the sample means is different from the rest.
B. At the 5% significance level there is no evidence that at least one of the population means is different from the rest.
C. At the 5% significance level there is no evidence that at least one of the sample means is different from the rest.
D. At the 5% significance level there is evidence that at least one of the population means is different from the rest.

_______ 19. (3 points) Choose the correct statement

A. At the 5% significance level population means for neighborhoods 1 and 2 are significantly different because the 95% confidence interval contains the zero value.
B. At the 5% significance level population means for neighborhoods 1 and 3 are not significantly different because the 95% confidence interval does not contain the zero value.
C. At the 5% significance level population means for neighborhoods 2 and 3 are not significantly different because the 95% confidence interval contains the zero value.

ANOVA

RecyHouseDay

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	.222	2	.111	3.052	.049
Within Groups	8.445	232	.036		
Total	8.667	234			

Multiple Comparisons

Dependent Variable: RecyHouseDay

Bonferroni

(I) neighb	(J) neighb	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
1.00	2.00	-.03871	.03066	.624	-.1126	.0352
	3.00	-.07475*	.03026	.043	-.1477	-.0018
2.00	1.00	.03871	.03066	.624	-.0352	.1126
	3.00	-.03604	.03056	.718	-.1097	.0377
3.00	1.00	.07475*	.03026	.043	.0018	.1477
	2.00	.03604	.03056	.718	-.0377	.1097

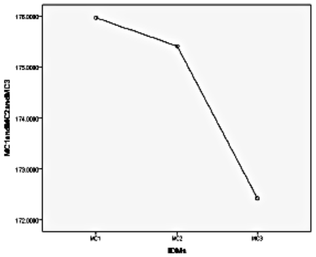
*. The mean difference is significant at the 0.05 level.

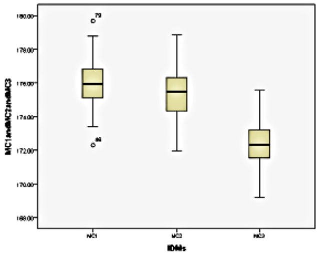
Heights were collected out of simple random samples of male citizens from three different countries MC1, MC2 and MC3. The aim of the study was to assess whether there is significant evidence (at the 5% significance level) that not all the population male heights are the same among these three countries. Use the outputs from SPSS to answer questions 23-26 in the next page. Write the correct letter answer on the line next to the question's number.

Descriptives

MC1andMC2andMC3

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean: Lower Bound	95% Confidence Interval for Mean: Upper Bound	Minimum	Maximum
1.00	100	175.9698	1.25596	.12560	175.7206	176.2191	172.31	179.69
2.00	100	175.4095	1.38964	.13896	175.1338	175.6853	171.97	178.88
3.00	100	172.4266	1.29774	.12977	172.1691	172.6841	169.20	175.55
Total	300	174.6020	2.03615	.11756	174.3706	174.8333	169.20	179.69





ANOVA

MC1andMC2andMC3

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	725.551	2	362.775	209.589	.000
Within Groups	514.075	297	1.731		
Total	1239.626	299			

Multiple Comparisons

Dependent Variable: MC1andMC2andMC3

Bonferroni

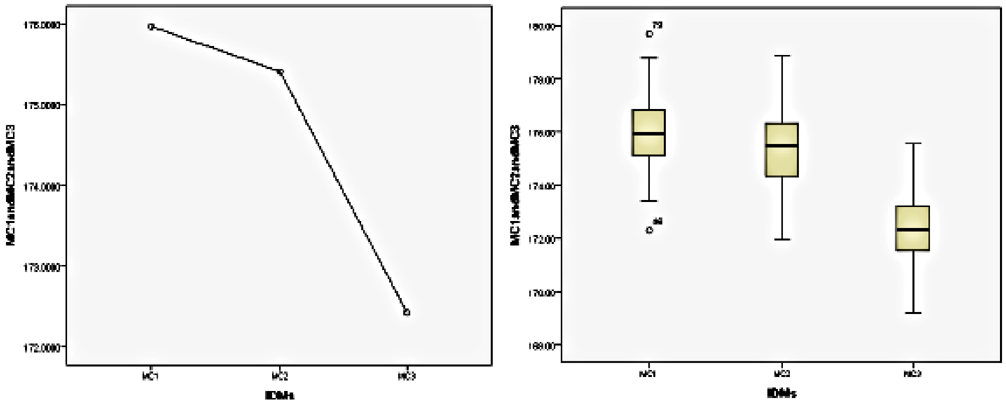
(I) NIDMs	(J) NIDMs	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval: Lower Bound	95% Confidence Interval: Upper Bound
1.00	2.00	.56033*	.18606	.008	.1124	1.0083
	3.00	3.54326*	.18606	.000	3.0953	3.9912
2.00	1.00	-.56033*	.18606	.008	-1.0083	-.1124
	3.00	2.98293*	.18606	.000	2.5350	3.4309
3.00	1.00	-3.54326*	.18606	.000	-3.9912	-3.0953
	2.00	-2.98293*	.18606	.000	-3.4309	-2.5350

*. The mean difference is significant at the 0.05 level.

_______ 23. (3 points) Based solely on the side-by-side boxplot and means plot we can say

A. There appears to be no difference between the sample averages of male heights of countries MC1, MC2 and MC3 because all the boxplots overlap.

B. There appears to be difference between the sample averages of male heights of countries MC1 and MC2 relative to male heights of country MC3 because (in the boxplots) the boxes of countries MC1 and MC2 do not overlap with the box of country MC3.



_______ 24. (3 points) We can trust the results of ANOVA test

A. Because 1.3896<2*1.25596 assuming that the data sets from the three states approximately follow a Normal distribution.
B. Because 2.03615<2*1.25596 assuming that the data sets from the three states approximately follow a Normal distribution.

Descriptives

MC1andMC2andMC3

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
1.00	100	175.9698	1.25596	.12560	175.7206	176.2191	172.31	179.69
2.00	100	175.4095	1.38964	.13896	175.1338	175.6853	171.97	178.88
3.00	100	172.4266	1.29774	.12977	172.1691	172.6841	169.20	175.55
Total	300	174.6020	2.03615	.11756	174.3706	174.8333	169.20	179.69

_______ 25. (3 points) At the 5% significance level we can conclude that

A. There is significant evidence that each pair of population averages of male heights are different. We can conclude this just by looking at the ANOVA output where F has a very large value and the corresponding Sig. is zero.
B. There is significant evidence that each pair of population averages of male heights are different. We can conclude this because all Sigs. in the Bonferroni Multiple Comparisons test are less than 0.05.

ANOVA

MC1andMC2andMC3

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	725.551	2	362.775	209.589	.000
Within Groups	514.075	297	1.731		
Total	1239.626	299			

Multiple Comparisons

Dependent Variable: MC1andMC2andMC3

Bonferroni

(I) NIDMs	(J) NIDMs	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
1.00	2.00	.56033*	.18606	.008	.1124	1.0083
	3.00	3.54326*	.18606	.000	3.0953	3.9912
2.00	1.00	-.56033*	.18606	.008	-1.0083	-.1124
	3.00	2.98293*	.18606	.000	2.5350	3.4309
3.00	1.00	-3.54326*	.18606	.000	-3.9912	-3.0953
	2.00	-2.98293*	.18606	.000	-3.4309	-2.5350

*. The mean difference is significant at the 0.05 level.

26. (3 points) Calculate the percentage of variation explained by the ANOVA model (R^2). Give your answer with two decimal places. SHOW YOUR WORK. (1 point for the correct numerator, 1 point for the correct denominator, 1 point for the correct answer).

ANOVA

MC1andMC2andMC3

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	725.551	2	362.775	209.589	.000
Within Groups	514.075	297	1.731		
Total	1239.626	299			